King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 8 Quiz III(B) (Term 163)

Name : .................................... ID #......KEY.................. Serial #: .......

1. If $f(x) = \dfrac{1+\tanh x}{1-\tanh x}$, then $f\left(\dfrac{1}{2}\right) =$

a) $2e$

b) $\ln 2$

c) $e$

d) $-\ln 2$

e) $2$

$$f(x) = \frac{1+\tanh x}{1-\tanh x} = \frac{1+\frac{e^x-e^{-x}}{e^x+e^{-x}}}{1-\frac{e^x-e^{-x}}{e^x+e^{-x}}}$$

$$= \frac{2e^x}{2e^{-x}} = e^{2x}$$

$$\Rightarrow f\left(\tfrac{1}{2}\right) = e$$

2. If $L(x)$ is the linearization of $f(x) = 1 + \ln(1-2x)$ near $a = 0$, then $L(-1) =$

a) $0$

b) $2$

c) $-3$

d) $-2$

e) $3$

$$f'(x) = \frac{-2}{1-2x}$$

$$L(x) = f(0) + f'(0)(x-0)$$

$$= 1 - 2x$$

$$\Rightarrow L(-1) = 3$$

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3. The radius of a cone was measured and found to be $3\,cm$ with a possible relative error of $\frac{0.03}{3}$. If the height of the cone is measured to be triple of the radius, then the relative error of the volume of the cone is: $h = 3r$

(Hint: $V = \frac{1}{3}\pi r^2 h$)

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$$

$$\Rightarrow dV = 3\pi r^2\, dr$$

relative error $\frac{dV}{V} = \frac{3\pi r^2 dr}{\pi r^3} = 3\frac{dr}{r}$

$$= 3\left(\frac{0.03}{3}\right) = 0.03$$

note $\frac{dr}{r} = \frac{0.03}{3}$

a) 0.01

b) 0.03

c) $(0.01)\,\pi$

d) $(0.03)\,\pi$

e) $3\,\pi$

4. If $C$ is a number that satisfies the conclusion on the Mean Value Theorem when applied to $f(x) = \ln x$ on $[1, e]$, then $C =$

$$f'(c) = \frac{f(e)-f(1)}{e-1} = \frac{1}{e-1}$$

$$\Rightarrow \frac{1}{c} = \frac{1}{e-1} \Rightarrow c = e-1$$

a) $\frac{e}{2}$

b) $\frac{1}{e-2}$

c) $e - 1$

d) $\frac{e+1}{2}$

e) $\frac{4}{e+1}$

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5. $\lim_{x\to 0^+} (1 - 4\sin(3x))^{5\cot(9x)} =$

a) $e^{-20}$

b) $e^{-20/3}$

c) $e^{-5/4}$

d) $e^{-15/14}$

e) 1

$$y = (1 - 4\sin(3x))^{5\cot(9x)}$$

$$\Rightarrow \ln y = 5\cot(9x)\ln(1-4\sin(3x))$$

$$\Rightarrow \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{5\ln(1-4\sin(3x))}{\tan(9x)} \quad \frac{0}{0}$$

$$\overset{LHR}{=} \lim_{x\to 0^+} \frac{\frac{5(-12\cos 3x)}{1-4\sin(3x)}}{9\sec^2(9x)} = \frac{-60}{9} = -\frac{20}{3}$$

$$\Rightarrow \lim_{x\to 0^+} y = e^{-20/3}$$

6. The slant asymptote of $f(x) = e^x + x + 1$ is

a) $y = x$

b) $y = 2x + 1$

c) $y = x + 1$

d) $y = -2x + 1$

e) None of them

Note $\lim_{x\to\infty} e^x = \infty$ but $\lim_{x\to -\infty} e^x = 0$

We are looking for m and b such that

$$\lim_{x\to\pm\infty} [f(x) - (mx+b)] = 0$$

Since $\lim_{x\to\infty} e^x = \infty \Rightarrow \lim_{x\to\infty} [e^x + (1-m)x + (1-b)] = \infty$

but $\lim_{x\to-\infty} [e^x + (1-m)x + (1-b)] = 0$ if $1-m=0$ & $1-b=0$

$\Rightarrow m=1$ & $b=1$

$\Rightarrow y = x+1$ is a slant asymptote

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7. Suppose that $3 \leq f'(x) \leq 5$ for all values of $x$. Then $a \leq f(8) - f(2) \leq b$ where $b - a$ is equal to

Since f(x) satisfy the MVT on [2, 8]

$\Rightarrow$ There exists $c \in (2, 8)$ s.t.

$$\frac{f(8) - f(2)}{8 - 2} = f'(c)$$

a) 10
b) 8
c) 6
d) 4
e) 12

$$\Rightarrow 3 \leq \frac{f(8) - f(2)}{6} = f'(c) \leq 5$$

$$\Rightarrow 18 \leq f(8) - f(2) \leq 30 \Rightarrow a = 18 \;\&\; b = 30$$

$$\Rightarrow b - a = 30 - 18 = 12$$

8. $\sin^{-1} x + \cos^{-1} x =$

$$f(x) = \sin^{-1} x + \cos^{-1} x$$

$$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

$\Rightarrow f(x)$ is constant

a) $\frac{\pi}{3}$
b) 1
c) $\frac{\pi}{2}$
d) 0
e) None of above

let $x = 0$ $\Rightarrow \sin^{-1} x + \cos^{-1} x = 0 + \frac{\pi}{2}$

$$= \frac{\pi}{2}$$

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9. The absolute maximum of $f(x) = xe^{-x^2/8}$ over $[-1, 4]$ is

$$f'(x) = e^{-x^2/8} + x * \left(-\frac{x}{4}\right) e^{-x^2/8}$$

$$= e^{-x^2/8}\left[1 - x^2/4\right] = 0$$

a) $\frac{4}{\sqrt{e}}$

b) $\frac{-1}{\sqrt[8]{e}}$

c) $\frac{2}{\sqrt{e}}$

d) $\frac{4}{\sqrt[8]{e}}$

e) $2\sqrt{e}$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$x = -2 \notin [-1, 4]$ ×

$x = 2 \in [-1, 4]$ ✓

check

$f(-1) = -e^{-1/8}$

$f(2) = 2e^{-1/2}$

$f(4) = 2e^{-2}$

Note $f(2) > f(4)$

$\Rightarrow$ Abs. Max. $\frac{2}{\sqrt{e}}$

10. The function $f(x) = \dfrac{\sqrt{1-x^2}}{x}$ is

$D_f$: $x \neq 0$

$1 - x^2 \geq 0 \Rightarrow x^2 \leq 1$

$\Rightarrow -1 \leq x \leq 1$

$x \neq 0$

a) increasing on $(-1, 0)$ and on $(0, 1)$

b) decreasing on $(-1, 0)$ and on $(0, 1)$

c) increasing on $(-1, 0)$ and decreasing on $(0, 1)$

d) increasing on $(0, 1)$ and decreasing on $(-1, 0)$

e) decreasing on $(-\infty, \infty)$

$$f'(x) = \frac{\sqrt{1-x^2} - x * \frac{-2x}{2\sqrt{1-x^2}}}{x^2} = \frac{1 - x^2 + x^2}{x^2\sqrt{1-x^2}} = \frac{1}{x^2\sqrt{1-x^2}}$$

$f'(x) \geq 0$ for all $x \in D_f$

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11. The polynomial $f(x) = 1 + 2x + 6x^2 - x^4$ is
(CD:concave downward, CU: concave upward)

$f'(x) = 2 + 12x - 4x^3$

$f''(x) = 12 - 12x^2$
$= 12(1-x)(1+x)$

a) CD on $(-\infty, 1)$; CU on $(1, \infty)$
b) CD on $(-\infty, -1)$; and $(1, \infty)$; CU on $(-1, 1)$
c) CD on $(-\infty, 2)$ and $(2, \infty)$; CU on $(2, \infty)$
d) CD on $(-\infty, \infty)$
e) CD on $(-3, 3)$; CU on $(-\infty, -3)$ and $(3, \infty)$

$-$    $+$    $-$

CD $-1$ CU $1$ CD

12. The sum of the critical numbers os $f(x) = (x-1)^{3/5}(4-x)$

a) $\frac{17}{8}$
b) $\frac{-9}{8}$
c) $0$
d) $\frac{3}{7}$
e) $\frac{25}{8}$

$f'(x) = \frac{3}{5}(x-1)^{-2/5}(4-x) - (x-1)^{3/5}$

$= (x-1)^{-2/5}\left[\frac{3}{5}(4-x) - (x-1)\right]$

$= (x-1)^{-2/5}\left[\frac{17}{5} - \frac{8}{5}x\right] = \frac{(x-1)^{-2/5}}{5}[17 - 8x]$

$= \dfrac{17 - 8x}{5(x-1)^{2/5}}$

$f'(x) = 0 \Rightarrow x = 17/8 \in D_f$

$f'(x)$ DNE $\Rightarrow x = 1 \in D_f$

$\Rightarrow$ Sum $= \frac{17}{8} + 1 = \frac{25}{8}$

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13. If $f(x) = \tan^{-1}(\sinh x)$, then $f'(x)$ is

a) $\operatorname{sech} x$

b) $\operatorname{csch} x$

c) $\tanh x$

d) $\coth x$

e) $\cosh x$

$$f'(x) = \frac{1}{1+\sinh^2 x} * \cosh x$$

$$= \frac{1}{\cosh^2 x} * \cosh x = \operatorname{sech} x$$