King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 8 Quiz II(A) (Term 163)

Name : KEY **ID** #........................ Serial #: ....

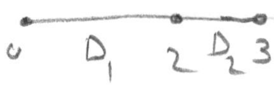
1. A particle moves according to a law of motion $s = f(t) = t^3 - 9t^2 + 24t$, $t > 0$, where t is measured in seconds and s in meters.

(a) Find the total distance travelled during the first 3 seconds.

$V(t) = f'(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$

$= 3(t-2)(t-4)$

$D_1 = |f(2) - f(0)|$



$= |20 - 0| = 20$

$D_2 = |f(3) - f(2)| = |18 - 20| = 2$

$D = D_1 + D_2 = 20 + 2 = 22$ m

(b) Find the acceleration when its velocity is equal to 24.

$V(t) = 24 \Rightarrow 3t^2 - 18t + 24 = 24 \Rightarrow 3t(t-6) = 0$

$\Rightarrow t = 6$ s

Now, $a(t) = 6t - 18$

$a(6) = 36 - 18 = 18$ m^2/s

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2. Find $f'(x)$ if $f(x) = (\ln x)^{\tan x}$

$y = (\ln x)^{\tan x} \Rightarrow \ln y = (\tan x) \ln(\ln x)$

Diff. w.r.t. $x \Rightarrow \frac{y'}{y} = \sec^2 x \ln(\ln x) + \frac{\tan x}{x \ln x}$

$\Rightarrow y' = (\ln x)^{\tan x} \left[\sec^2 x \ln(\ln x) + \frac{\tan x}{x \ln x} \right]$

3. If $y = \sqrt[3]{\dfrac{x(x+1)(x+2)}{(x+3)(x+4)(x+5)}}$, then find $y'|_{x=1}$.

$\ln y = \frac{1}{3}(\ln x + \ln(x+1) + \ln(x+2) - \ln(x+3) - \ln(x+4) - \ln(x+5))$

Diff. w.r.t. $x \Rightarrow$

$\frac{y'}{y} = \frac{1}{3}\left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3} - \frac{1}{x+4} - \frac{1}{x+5} \right]$

At $x = 1 \Rightarrow y = \sqrt[3]{\frac{1(2)(3)}{4(5)(6)}} = \frac{1}{\sqrt[3]{20}}$

$\Rightarrow y'\big|_{x=1} = \frac{1}{3\sqrt[3]{20}} \left[1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6} \right] = \frac{1}{3\sqrt[3]{20}} \left[\frac{60+30+20-15-12-10}{60} \right]$

$= \frac{73}{180\sqrt[3]{20}}$

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4. Find $\lim_{x\to\infty}\left(1+\frac{5}{3x}\right)^x$ $= \lim_{x\to\infty}\left(1+\frac{5/3}{x}\right)^x = e^{5/3}$

5. If $x^3 - y^3 = 7$, then find y'' in its simplest form.

Diff. w.r.t. x $\Rightarrow 3x^2 - 3y^2y' = 0 \Rightarrow y' = \frac{x^2}{y^2}$

$\Rightarrow y'' = \frac{2xy^2 - 2x^2yy'}{y^4} = \frac{2xy^2 - \frac{2x^4}{y}}{y^4} = \frac{2xy^3 - 2x^4}{y^5}$

$= \frac{2x(y^3 - x^3)}{y^5} = \frac{-14x}{y^5}$

6. Let $f(x) = 1 + 2x - x^2$, $x \le 1$. Then find $\frac{df^{-1}}{dx}|_{x=-2}$.

$\frac{df^{-1}}{dx}\Big|_{x=-2} = \frac{1}{f'(f^{-1}(-2))}$

To find $f^{-1}(-2)$, we need to find x such that

$-2 = 1 + 2x - x^2 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = -1$ or $\cancel{x = 3}$ $\Rightarrow f^{-1}(-2) = -1$

Also, $f'(x) = 2 - 2x$

$\Rightarrow \frac{df^{-1}}{dx}\Big|_{x=-2} = \frac{1}{2+2} = \frac{1}{4}$

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7. Find $\dfrac{d^{19}}{dx^{19}}\,(x \sin x)$

Note: $19 = 4(4) + 3$

$y = x \sin x$

$y' = \sin x + x \cos x$

$y'' = \cos x + \cos x - x \sin x = 2 \cos x - x \sin x$

$y''' = -2 \sin x - \sin x - x \cos x = -3 \sin x - x \cos x$

$y^{(4)} = -3 \cos x - \cos x + x \sin x = -4 \cos x + x \sin x$

$\vdots$

$y^{(19)} = -19 \sin x - x \cos x$

8. If $f(x) = \tan^2\left(x^2 - \dfrac{3\pi}{4}\right)$, then find $f'(\sqrt{\pi})$

$f'(x) = 2 \tan\left(x^2 - \frac{3\pi}{4}\right) * \sec^2\left(x^2 - \frac{3\pi}{4}\right) * 2x$

$\Rightarrow f'(\sqrt{\pi}) = 2 \tan \frac{\pi}{4} \sec^2 \frac{\pi}{4} * 2\sqrt{\pi}$

$= 8\sqrt{\pi}$

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9. If $F(x) = g(x) \cdot f(g(x))$, where $f(3) = 4$, $g(2) = 3$, $f'(3) = 5$, $g'(2) = 10$, then find $F'(2)$.

$F'(x) = g'(x)\, f(g(x)) + g(x)\, f'(g(x))\, g'(x)$

$\Rightarrow F'(2) = g'(2)\, f(g(2)) + g(2)\, f'(g(2))\, g'(2)$

$= 10\, f(3) + 3\, f'(3) * 10 = 40 + 150 = 190$

10. If $y = x^2 \sin^{-1}(x^2) + \sqrt{1 - x^4}$, then find y' in its simplest form.

$y' = 2x \sin^{-1}(x^2) + x^2 * \frac{2x}{\sqrt{1-x^4}} + \frac{-4x^3}{2\sqrt{1-x^4}}$

$= 2x \sin^{-1}(x^2)$

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11. If $y = \sin 2x - \cos 2x$, then find $y^{(4)}(0)$.

$y' = 2\cos 2x + 2\sin 2x$

$y'' = -4\sin 2x + 4\cos 2x$

$y''' = -8\cos 2x - 8\sin 2x$

$y^{(4)} = 16\sin 2x - 16\cos 2x$

$\Rightarrow y^{(4)}(0) = -16$

12. Find $\lim\limits_{x\to\pi} \dfrac{\sin(\sin x)}{\tan x} = \lim\limits_{x\to\pi} \dfrac{\sin(\sin x)}{\frac{\sin x}{\cos x}} = \lim\limits_{x\to\pi} \cos x \cdot \dfrac{\sin(\sin x)}{\sin x}$

Note for $\lim\limits_{x\to\pi} \dfrac{\sin(\sin x)}{\sin x}$, if we let $y = \sin x$, then

$\lim\limits_{x\to\pi} \dfrac{\sin(\sin x)}{\sin x} = \lim\limits_{y\to 0} \dfrac{\sin y}{y} = 1$ (since if $x \to \pi$, then $y \to 0$)

$\therefore \lim\limits_{x\to\pi} \dfrac{\sin(\sin x)}{\tan x} = \lim\limits_{x\to\pi} \cos x \cdot \lim\limits_{x\to\pi} \dfrac{\sin(\sin x)}{\sin x}$

$= (-1) \cdot 1 = -1.$

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13. Find the equation of the normal line to the graph of the curve $y = \dfrac{1+\sin x}{x+\cos x}$ at $\left(\pi, \dfrac{1}{\pi - 1}\right)$.

$$y' = \frac{\cos x\,(x+\cos x) - (1-\sin x)(1+\sin x)}{(x+\cos x)^2} = \frac{x\cos x}{(x+\cos x)^2}$$

$$\therefore m_T = y'\Big|_{x=\pi} = \frac{\pi(-1)}{(\pi-1)^2} = \frac{-\pi}{(\pi-1)^2}$$

$$\Rightarrow m_N = \frac{(\pi-1)^2}{\pi}$$

Equation $\quad y - \dfrac{1}{\pi-1} = \dfrac{(\pi-1)^2}{\pi}(x-\pi)$

14. If $f(t) = 3^{\log_2 t} + \log_2(3^t)$, then find $f'(2)$.

$$f(t) = 3^{\log_2 t} + t\log_2 3 \Rightarrow f'(t) = 3^{\log_2 t}\,\ln 3 \cdot \frac{1}{t\ln 2} + \log_2 3$$

$$\Rightarrow f'(2) = \frac{3\ln 3}{2\ln 2} + \frac{\ln 3}{\ln 2} = \frac{5\ln 3}{2\ln 2}$$

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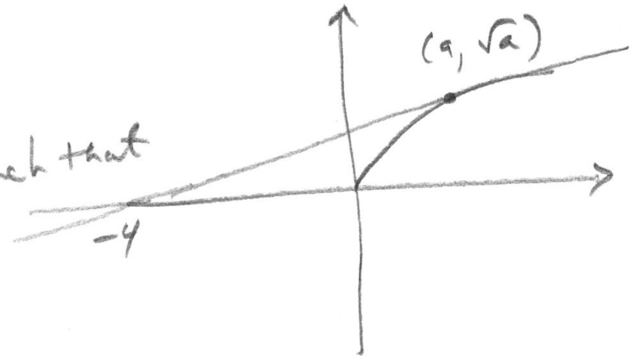
15. Find an equation for the tangent line to the curve $y = \sqrt{x}$ that passes through the point $(-4, 0)$.

$$y' = \frac{1}{2\sqrt{x}}$$



we need to find a such that

$$m = \frac{\sqrt{a} - 0}{a + 4} = \frac{1}{2\sqrt{a}}$$

$$\Rightarrow 2a = a + 4 \Rightarrow a = 4$$

$\Rightarrow m = \frac{1}{4}$ and the point of tangency is $(4, 2)$

Equation: $y - 2 = \frac{1}{4}(x - 4)$

$$\Rightarrow y = \frac{1}{4}x + 1$$

16. If $y = u^2 - 1$ and $u = e^{2x} + \ln x$ then find $\dfrac{dy}{dx}$ at $x = 1$.

$\frac{dy}{du} = 2u$ & $\frac{du}{dx} = 2e^{2x} + \frac{1}{x}$

At $x = 1 \Rightarrow u = e^2$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{dy}{du}\right|_{x=1} \cdot \left.\frac{du}{dx}\right|_{x=1}$$

$$= \left.\frac{dy}{du}\right|_{u=e^2} \cdot \left.\frac{du}{dx}\right|_{x=1} = 2e^2(2e^2 + 1)$$

$$= 4e^4 + 2e^2$$