King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 8 Quiz I(B) (Term 163)

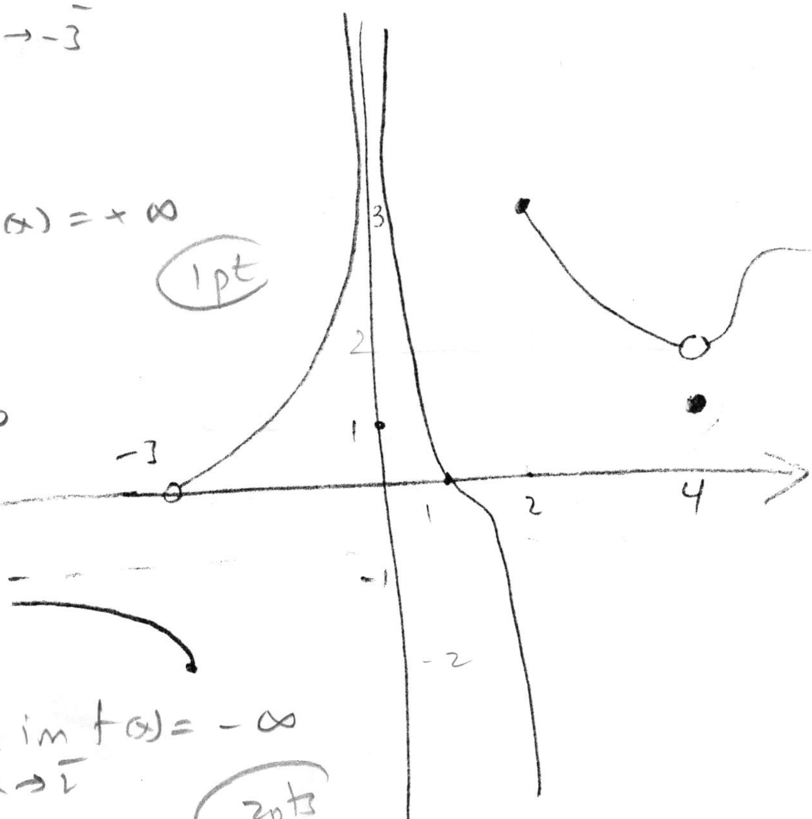
Name : ......KEY...... ID #...................................... Serial #: ....

10 pts

1. Evaluate the limit if it exists. If it does not exist, explain why? Use the symbols ∞ or $-\infty$ when needed

(i) $\lim_{x\to -3} f(x)$ DNE because 2pts

$\lim_{x\to -3^+} f(x) = 0$ & $\lim_{x\to -3^-} f(x) = -2$



(ii) $\lim_{x\to 0} f(x) = +\infty$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = +\infty$ 1pt

(iii) $\lim_{x\to 1} f(x) = f(1) = 0$

1pt

(iv) $\lim_{x\to 2} f(x)$ DNE

$\lim_{x\to 2^+} f(x) = 3$ & $\lim_{x\to 2^-} f(x) = -\infty$

2pts

(v) $\lim_{x\to 4} f(x) = 2$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = 2$ 1pt

(vi) $\lim_{x\to 4^-} (2f(x) + 3g(x))$ where $g(x) = [x-1]$ and $[x]$ is the greatest integer $\leq x$.

$= 2\lim_{x\to 4^-} f(x) + 3\lim_{x\to 4^-} g(x) = 2(2) + 3(2) = 10$

1pt 1pt

1pt

10 pts

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25pts

2. Evaluate the limit or show that it does not exist:

(i) $\lim_{x\to 3^-} \frac{|x^2-7x+12|}{2x-6} = \lim_{x\to 3^-} \frac{|(x-4)(x-3)|}{2(x-3)}$ 1pt

$= \lim_{x\to 3^-} \frac{|x-4|\,|x-3|}{2(x-3)} = \lim_{x\to 3^-} \frac{(-(x-4))(-\cancel{(x-3)})}{2\cancel{(x-3)}}$ 2pts

$= \frac{+(3-4)}{2} = -\frac{1}{2}$ 1pt

(ii) $\lim_{x\to 0^+} [1-\sin x]$ where $[x]$ is the greatest integer $\leq x$.

$= 0$

3pts

(iii) $\lim_{x\to 2} f(x)$ where $f(x)\begin{cases} x-1 & x\leq 2 \\ 2x-4 & x>2 \end{cases}$

$\lim_{x\to 2^+} f(x) = 2(2)-4 = 0$ 1pt

$\neq \Rightarrow \lim_{x\to 2} f(x)$ DNE

$\lim_{x\to 2^-} f(x) = 2-1 = 1$

1pt 1pt

10pts

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(iv) $\lim_{x\to 0} \dfrac{\sqrt{4-x}-2}{\sqrt{1-x}-1} \cdot \dfrac{\sqrt{4-x}+2}{\sqrt{4-x}+2} \cdot \dfrac{\sqrt{1-x}+1}{\sqrt{1-x}+1}$ 2pts

$= \lim_{x\to 0} \dfrac{(4-x-4)(\sqrt{1-x}+1)}{(1-x-1)(\sqrt{4-x}+2)} = \lim_{x\to 0} \dfrac{\cancel{-x}(\sqrt{1-x}+1)}{\cancel{-x}(\sqrt{4-x}+2)}$ 3pts

$= \dfrac{2}{4} = \dfrac{1}{2}$ 1pt

2pts

(v) $\lim_{u\to -1} \dfrac{u^3+1}{u^2-1} = \lim_{u\to -1} \dfrac{\cancel{(u+1)}(u^2-u+1)}{\cancel{(u+1)}(u-1)} = \dfrac{1+1+1}{-1-1}$

$= \dfrac{-3}{2}$ 2pts

10pts

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(vi) $\lim_{x \to 0} x^4 \sin \frac{\pi}{x}$

1pt

We have $-1 \leq \sin \frac{\pi}{x} \leq 1$

1pt

$-x^4 \leq x^4 \sin \frac{\pi}{x} \leq x^4$

as $x \to 0$: $-x^4 \to 0$, $x^4 \to 0$

ST: $x^4 \sin \frac{\pi}{x} \to 0$

3pts

$\Rightarrow \lim_{x \to 0} x^4 \sin \frac{\pi}{x} = 0$

10pts

3. Find all vertical asymptotes for the function

$$f(x) = \frac{(x-2)\ln(x-1)}{x^2-4}$$

2pts

possible V.A. $x = 1$ & $x = \pm 2$

Domain: $x > 1$ & $x \neq \pm 2 \Rightarrow D_f: (1,2) \cup (2,\infty)$

1pt

$x = -2$ excluded 1pt

$x = 1$ $\lim_{x \to 1^+} \frac{\cancel{(x-2)} \ln(x-1)}{\cancel{(x-2)}(x+2)} = -\infty$

3pts

$\Rightarrow x = 1$ is a V.A.

2pts

$x = 2$ $\lim_{x \to 2^+} \frac{\ln(x-1)}{x+2} = \frac{0}{4} = 0$

1pt

$\Rightarrow$ The only V.A. is $x = 1$.

15pts

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7pts 4. Use the graph of $y = \sqrt{19-x} - 3$ to find the largest number $\delta > 0$ such that if $0 < |x - 10| < \delta$, then $|\sqrt{19-x} - 3| < 1$. 2pts

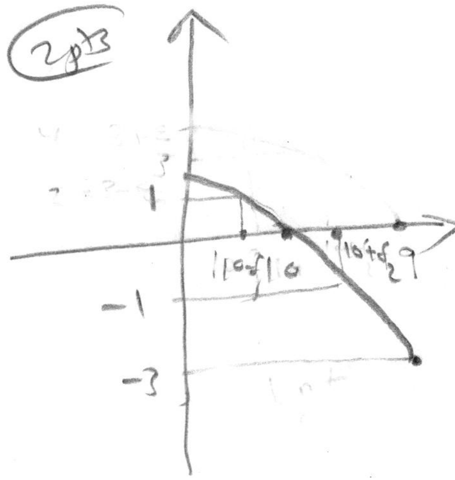
$f(x) = \sqrt{19-x} - 3$ & $L = 0$ & $a = 10$ & $\varepsilon = 1$

$f(10-\delta_1) = 1$ & $f(10+\delta_1) = -1$. 2pts

$\Rightarrow \sqrt{9+\delta_1} - 3 = 1$ & $\sqrt{9-\delta_2} - 3 = -1$ 2pts

$\Rightarrow 9 + \delta_1 = 16 \Rightarrow \delta_1 = 7$ & $9 - \delta_2 = 4 \Rightarrow \delta_2 = 5$

$\delta = \min\{\delta_1, \delta_2\} = 5$. 1pt



8pts 5. If $\lim_{x\to 1} \dfrac{\sqrt{ax+b} - a - 2}{x-1} = 1$, then find the values of a and b.

First, for the limit to be exist, we need $\sqrt{ax+b-a} - 2$ to be equal to 0 at $x=1$. That means

$\sqrt{a+b-a} - 2 = 0 \Rightarrow \sqrt{b} - 2 = 0 \Rightarrow b = 4$. 2pts 1pt 1pt 1pt

$\Rightarrow \lim_{x\to 1} \dfrac{\sqrt{ax+4-a} - 2}{x-1} \cdot \dfrac{\sqrt{ax+4-a}+2}{\sqrt{ax+4-a}+2} = \lim_{x\to 1} \dfrac{ax+4-a-4}{(x-1)(\sqrt{ax+4-a}+2)}$

$= \lim_{x\to 1} \dfrac{a(x-1)}{(x-1)(\sqrt{ax+4-a}+2)} = \dfrac{a}{\sqrt{4}+2} = \dfrac{a}{4} = 1 \Rightarrow a = 4$. 1pt

2pts

15pts