King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 7 Quiz III(A) (Term 163)

Name : .... ID #.......KEY.............. Serial #: ....

1. Using linear approximation, the value of $(64.018)^{2/3}$ is approximately equal to:

$f(x) = x^{2/3}$ $\quad a = 64 \quad$ $f'(x) = \frac{2}{3} x^{-1/3}$

a) 16.01
b) 16.003
c) 4.003
d) 16.018
e) 12.0.2

$\Rightarrow L(x) = f(a) + f'(a)(x-a)$
$= 16 + \frac{1}{6}(x - 64)$

$\Rightarrow (64.018)^{2/3} \approx 16 + \frac{1}{6}(0.018) = 16.003$

2. The radius of a circular disk is measured to be $5\,cm$ with a maximum error in measurement of $0.1\,cm$. Using differentials, the maximum error in calculating circumference of the circular disk is (in cm)

a) $\frac{\pi}{10}$
b) π
c) $\frac{\pi}{5}$
d) $\frac{\pi}{2}$
e) $\frac{\pi}{50}$

$C = 2\pi r$
$\Rightarrow dC = 2\pi\, dr = 2\pi * (0.1)$
$= 0.2\pi = \frac{\pi}{5}$

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3. If $\cosh x = \frac{5}{3}$ and $x < 0$, then $3 \sinh x + 5 \tanh x$ is equal to

a) -6
b) 8
c) 6
d) 0
e) -8

Start from $\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow \sinh^2 x = \cosh^2 x - 1 = \frac{25}{9} - 1 = \frac{16}{9}$

$\Rightarrow \sinh x = \frac{-4}{3}$ since $x<0$

$\Rightarrow \tanh x = \frac{\sinh x}{\cosh x} = \frac{-4}{3} * \frac{3}{5} = \frac{-4}{5}$

$\Rightarrow 3 \sinh x + 5 \tanh x = -4 + (-4) = -8$

4. The curve $y = \cosh(\ln x) + 4x$ has a horizontal tangent line at $x =$

a) $\frac{1}{6}$
b) $-\frac{1}{3}$
c) -3
d) $\frac{1}{3}$
e) 3

$y' = \sinh(\ln x) * \frac{1}{x} + 4 = 0$

$\Rightarrow \frac{e^{\ln x} - e^{-\ln x}}{2} * \frac{1}{x} + 4 = 0$

$\Rightarrow \frac{x - \frac{1}{x}}{2} * \frac{1}{x} + 4 = 0$

$\Rightarrow \frac{x^2 - 1}{2x^2} + 4 = \frac{x^2 - 1 + 8x^2}{2x^2}$

$= \frac{9x^2 - 1}{2x^2} = 0$

$\Rightarrow 9x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3}$

$x = -\frac{1}{3}$ ✗

$x = \frac{1}{3}$ ✓

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5. The function $f(x) = \dfrac{x}{x^2+1}$ is increasing on

$$f'(x) = \frac{1.(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

a) $(-\infty, 0)$
b) $(0, \infty)$
c) $(-1, 1)$
d) $(-\infty, -1) \cup (1, \infty)$
e) $(-\infty, \infty)$

$-$ $+$ $-$

-1 1

6. If the function $f(x) = x^3 + 2ax^2 - 3bx + 1$ has an inflection point at $(1, 2)$, the $2a + b^3$ equals

$$f'(x) = 3x^2 + 4ax - 3b$$

a) -1
b) -2
c) 2
d) -4
e) 3

$$f''(x) = 6x + 4a$$

$$\Rightarrow f''(1) = 0 \quad \& \quad f(1) = 2$$

$$\Rightarrow 6 + 4a = 0 \Rightarrow a = -3/2$$

Also, $1 - 3 - 3b + 1 = 2$

$$\Rightarrow -3b = 3 \Rightarrow b = -1$$

$$\Rightarrow 2a + b^3 = -3 - 1 = -4$$

7. If $f(5) = \frac{-5}{2}$ and $f'(x) \geq \frac{-1}{2}$ for $3 \leq x \leq 5$, then the largest possible value of $f(3)$ is

a) $\frac{-2}{5}$

b) $\frac{-3}{2}$

c) $\frac{1}{2}$

d) $\frac{-1}{4}$

e) 0

Since $f(x)$ satisfy the MVT on $[3,5]$

$\Rightarrow$ There exists $c \in (3,5)$ s.t.

$$\frac{f(5)-f(3)}{5-3} = f'(c)$$

$$\Rightarrow \frac{-5/2 - f(3)}{2} \geq -\frac{1}{2}$$

$$\Rightarrow -\frac{5}{2} - f(3) \geq -1 \Rightarrow -f(3) \geq 3/2 \Rightarrow f(3) \leq -3/2$$

8. If c is number satisfying the conclusion of the Mean Value Theorem when applied to $f(x) = \tan^{-1} x$ on $[0,1]$, then $\pi\, c^2 =$

a) $\pi + 1$

b) 2π

c) $\pi - 2$

d) π

e) $4 - \pi$

$$f'(c) = \frac{1}{1+c^2}$$

$$\frac{f(1)-f(0)}{1-0} = \frac{\frac{\pi}{4} - 0}{1} = \frac{\pi}{4}$$

$$\overset{MVT}{\Rightarrow} \frac{1}{1+c^2} = \frac{\pi}{4} \Rightarrow 1+c^2 = \frac{4}{\pi}$$

$$\Rightarrow c^2 = \frac{4}{\pi} - 1$$

$$\Rightarrow \pi c^2 = 4 - \pi$$

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9. The sum of all critical numbers of the function

$$f(x) = \frac{(x-4)^2}{\sqrt[3]{x+1}}$$

is

$$f'(x) = \frac{2(x-4)(x+1)^{1/3} - (x-4)^2 * \frac{1}{3}(x+1)^{-2/3}}{(x+1)^{2/3}}$$

$$= \frac{(x-4)(x+1)^{-2/3}\left[2(x+1) - \frac{1}{3}(x-4)\right]}{(x+1)^{2/3}}$$

$$= \frac{(x-4)\left[6(x+1) - (x-4)\right]}{3(x+1)^{4/3}} = \frac{(x-4)(5x+10)}{3(x+1)^{4/3}}$$

a) 2
b) -1
c) -2
d) 1
e) 4

$f'(x) = 0 \Rightarrow x = 4$ & $x = -2$ $\qquad x = -1 \notin D_f$ X

$\Rightarrow$ Sum $= 4 - 2 = 2$

10. The sum of the absolute maximum value and the absolute minimum value of the function $f(x) = 2\sin x + \cos 2x$ on the interval $\left[0, \frac{\pi}{2}\right]$ is

a) $\frac{3}{2}$
b) 2
c) 3
d) $\frac{5}{2}$
e) $\frac{7}{2}$

$f'(x) = 2\cos x - 2\sin 2x = 0$

$\Rightarrow 2\cos x - 4\cos x \sin x = 0$

$\Rightarrow 2\cos x(1 - 2\sin x) = 0$

$\Rightarrow \cos x = 0$ & $\sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{2}$ & $\frac{\pi}{6}$

$\Rightarrow f(0) = 1$

$f(\frac{\pi}{6}) = 1 + \frac{1}{2} = 3/2$

$f(\frac{\pi}{2}) = 1$

Abs. max. $= 3/2$ & Abs. min. $= 1$

$\Rightarrow$ Sum $= 1 + 3/2 = 5/2$

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11. $\lim_{x \to 0} \frac{\sin^{-1}(2x)}{x} =$ $\frac{0}{0}$

a) equals 2
b) equals 1
c) does not exist
d) equals 0
e) equals -1

$$\Rightarrow \lim_{x \to 0} \frac{\sin^{-1}(2x)}{x} \overset{LHR}{=} \lim_{x \to 0} \frac{\frac{2}{\sqrt{1-(2x)^2}}}{1}$$

$$= 2$$

12. The slant asymptote of $y = \frac{2x^3 + 3x^2 + 10}{x^2 + 1}$ is

a) $y = 2x - 3$
b) $y = 2x + 1$
c) $y = 2x - 1$
d) $y = 2x + 3$
e) $y = 2x$

$$\begin{array}{r|l} & 2x + 3 \\ \hline x^2+1 & 2x^3 + 3x^2 \quad + 10 \\ & \mp 2x^3 \quad \mp 2x \\ \hline & 3x^2 - 2x + 10 \\ & 3x^2 \quad \mp 3 \\ \hline & -2x + 7 \end{array}$$

$\Rightarrow$ Slant Asymp. is

$y = 2x + 3$

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13. The value of the limit $\lim_{x\to 0^+} (1 - \sin x)^{1/x}$ equals

a) 0

b) $\frac{1}{\sqrt{e}}$

c) e

d) 1

e) $\frac{1}{e}$

$$y = (1 - \sin x)^{\frac{1}{x}}$$

$$\Rightarrow \ln y = \frac{1}{x} \ln(1 - \sin x)$$

$$\Rightarrow \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1-\sin x)}{x} \qquad \frac{0}{0}$$

$$\overset{LHR}{=} \lim_{x\to 0^+} \frac{\frac{-\cos x}{1-\sin x}}{1} = -1$$

$$\Rightarrow \lim_{x\to 0^+} y = e^{-1} = \frac{1}{e}$$