King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 7 Quiz II(B) (Term 163)

Name : ....KEY.... ID #.................................... Serial #: ....

1. A particle moves according to a law of motion $s = f(t) = \sin\left(\frac{\pi t}{2}\right)$, $t \geq 0$, where t is measured in seconds and s in meters. When is the particle slowing down in the first 6 seconds?

$$V = f'(t) = \cos\left(\frac{\pi t}{2}\right) \cdot \frac{\pi}{2} = \frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right)$$

$$a = v'(t) = f''(t) = -\frac{\pi^2}{4}\sin\left(\frac{\pi t}{2}\right)$$

$v(t)$		+ve	−ve	−ve	+ve	+ve	−ve
	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
$\frac{\pi t}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
$a(t)$		−ve	−ve	+ve	+ve	−ve	−ve

The particle is slowing down if

$0 < t < 1$, $2 < t < 3$ or $4 < t < 5$.

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2. Find $f'(x)$ if $f(x) = (\sin x)^{\csc x}$

$\ln y = \csc x \ \ln(\sin x)$

$\Rightarrow \frac{y'}{y} = -\csc x \cot x \ \ln(\sin x) + \csc x * \frac{\cos x}{\sin x}$

$\Rightarrow y' = (\sin x)^{\csc x} \left[-\csc x \cot x \ \ln(\sin x) + \csc x \cot x \right]$

3. Find the instantaneous rate of change of $f(x) = \dfrac{x^2 e^{\sqrt{x-1}}}{1-x}$ with respect to x at $x = 2$.

$y = \frac{x^2 e^{\sqrt{x-1}}}{1-x} \Rightarrow \ln y = 2 \ln x + \sqrt{x-1} - \ln(1-x)$

$\Rightarrow \frac{y'}{y} = \frac{2}{x} + \frac{1}{2\sqrt{x-1}} + \frac{1}{1-x}$

$x = 2 \Rightarrow y = \frac{4e}{-1} = -4e$

$\Rightarrow y'\big|_{x=2} = -4e\left[1 + \frac{1}{2} - 1\right] = -2e$

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4. Find $\lim_{x\to\infty}\left(1-\frac{2}{5x}\right)^x$ $= \lim_{x\to\infty}\left(1+\frac{-2/5}{x}\right)^x = e^{-2/5}$

5. If $x^2+4y^2=4$ then find y'' in its simplest form.

$\Rightarrow 2x + 8yy' = 0 \Rightarrow y' = \frac{-2x}{8y} = \frac{-x}{4y}$

$\Rightarrow y'' = \frac{-4y+4xy'}{16y^2} = \frac{-4y - \frac{4x^2}{4y}}{16y^2} = \frac{-16y^2-4x^2}{64y^3}$

$= \frac{-4(x^2+4y^2)}{64y^3} = \frac{-1}{4y^3}$

6. Let $f(x)=5x+3e^{7x}$. Then find $\frac{df^{-1}}{dx}|_{x=3}$.

$\frac{df^{-1}}{dx}\Big|_{x=3} = \frac{1}{f'(f(3))}$

To find $f^{-1}(3)$, we need to find x such that $f(x)=3$

$\Rightarrow 3 = 5x+3e^{7x} \Rightarrow x = 0 \Rightarrow f^{-1}(3) = 0$

Also, $f'(x) = 5+21e^{7x} \Rightarrow f'(0) = 5+21 = 26$

$\Rightarrow \frac{df^{-1}}{dx}\Big|_{x=3} = \frac{1}{26}$

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7. Evaluate $\lim_{x\to\frac{\pi}{4}} \dfrac{\cot x - 1}{x - \frac{\pi}{4}}$

If $y = f(x) = \cot x$, then $f'\left(\frac{\pi}{4}\right) = \lim_{x\to\frac{\pi}{4}} \dfrac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$

$= \lim_{x\to\frac{\pi}{4}} \dfrac{\cot x - 1}{x - \frac{\pi}{4}}$

Note $f'(x) = -\csc^2 x \Rightarrow f'\left(\frac{\pi}{4}\right) = -2$

$\therefore \lim_{x\to\frac{\pi}{4}} \dfrac{\cot x - 1}{x - \frac{\pi}{4}} = f'\left(\frac{\pi}{4}\right) = -2$

8. For $t > 0$, find $\dfrac{d}{dt}\left[\sin^{-1}\left(\dfrac{t-4}{t+4}\right)\right]$ in its simplest form.

$\dfrac{d}{dt}\left[\sin^{-1}\left(\dfrac{t-4}{t+4}\right)\right] = \dfrac{1}{\sqrt{1-\left(\frac{t-4}{t+4}\right)^2}} * \left[\dfrac{t+4-t+4}{(t+4)^2}\right]$

$= \dfrac{t+4}{\sqrt{(t+4)^2 - (t-4)^2}} \left[\dfrac{8}{(t+4)^2}\right] = \dfrac{8}{(t+4)\sqrt{t^2+8t+16-t^2+8t-16}}$

$= \dfrac{8}{(t+4)*4\sqrt{t}} = \dfrac{2}{\sqrt{t}(t+4)} = \dfrac{2\sqrt{t}}{t(t+4)}$

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9. If $h(x) = 2g(x) + f(\sqrt{g(x)})$, and $h'(-1) = 7$, $f'(3) = 18$, $g(-1) = 9$, then find $g'(-1)$.

$$\Rightarrow h'(x) = 2g'(x) + f'(\sqrt{g(x)}) * \frac{g'(x)}{2\sqrt{g(x)}}$$

$$\Rightarrow h'(-1) = 2g'(-1) + f'(\sqrt{g(-1)}) * \frac{g'(-1)}{2\sqrt{g(-1)}}$$

$$\Rightarrow 7 = 2g'(-1) + f'(3) * \frac{g'(-1)}{6}$$

$$\Rightarrow 7 = 5g'(-1) \Rightarrow g'(-1) = \frac{7}{5}$$

10. If $y = \cos(2 \ln x)$, then find $x^2y'' + xy'$ in its simplest form.

$$\Rightarrow y' = -\sin(2\ln x) * \frac{2}{x}$$

$$\Rightarrow y'' = -\cos(2\ln x) * \frac{4}{x^2} + \frac{2\sin(2\ln x)}{x^2}$$

$$\Rightarrow x^2y'' + xy' = -4\cos(2\ln x) + 2\sin(2\ln x) - 2\sin(2\ln x)$$

$$= -4\cos(2\ln x)$$

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11. If $f(x) = \sin(x) + \cos(x)$, then find $f^{(20)}(0) + f^{(21)}(0)$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

$f'''(x) = -\cos x + \sin x$

$f^{(4)}(x) = \sin x + \cos x$

$\vdots$

$f^{(20)}(x) + f^{(21)}(x) = \sin x + \cos x + \cos x - \sin x$

$= 2\cos x$

$\Rightarrow f^{(20)}(0) + f^{(21)}(x) = 2\cos 0 = 2$

12. Find $\lim_{t\to 0} \frac{3 \tan 2t - 5 \tan 3t}{7t \cos t + 4 \sin 5t} \div \frac{t}{t}$

$= \lim_{t\to 0} \frac{2 * 3 \frac{\tan 2t}{2t} - 3*5 \frac{\tan 3t}{3t}}{7 \cos t + 5*4 \frac{\sin 5t}{5t}}$

$= \frac{6 - 15}{7 + 20} = \frac{-9}{27} = \frac{-1}{3}$

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13. Find the equation of the tangent line to the curve
$y = \dfrac{1}{\sin x + \cos x}$ at $x = 0$.

$y = (\sin x + \cos x)^{-1}$

$\Rightarrow y' = -(\sin x + \cos x)^{-2} \cdot (\cos x - \sin x)$

$\Rightarrow y'\big|_{x=0} = -(1)^{-2}(1-0) = -1$

$x = 0 \Rightarrow y = 1 \Rightarrow$ point $(0, 1)$

Equation: $y - 1 = -1(x - 0) \Rightarrow y = -x + 1$

14. Find the slope of the tangent line to the graph of $y = 2^{3x} - \log_3(x+1)^5$ at $x = 1$.

$y = 2^{3x} + 5\log_3(x+1)$

$\Rightarrow y' = 2^{3x} \cdot \ln 2 \cdot 3 - \dfrac{5}{(x+1)\ln 3}$

$y'\big|_{x=1} = 8 \ln 8 - \dfrac{5}{2\ln 3}$

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15. If the normal line to the curve $x^2 - xy + y^2 = 1$ at $(1,1)$ intersects the curve at another point (a,b), then find $a+b$.

Diff. w.r.t. x $\Rightarrow 2x - y - xy' + 2yy' = 0$

At(1,1) $\Rightarrow 2 - 1 - y' + 2y' = 0 \Rightarrow m_T = y' = -1$

$\Rightarrow m_N = 1$

Equation of Normal line: $y - 1 = x - 1 \Rightarrow y = x$

Substitute in the curve to get $y^2 - y^2 + y^2 = 1 \Rightarrow y = \pm 1$

At $y = 1 \Rightarrow x = 1$ X

$y = -1 \Rightarrow x = -1 \Rightarrow a = -1$ and $b = -1$

$\Rightarrow a + b = -2$

16. If $z = \left(\frac{u-1}{u+1}\right)^2$ and $u = \frac{1}{x^2} - 1$ then find $\frac{dz}{dx}|_{x=-1}$

$\frac{dz}{du} = 2\left(\frac{u-1}{u+1}\right) * \left(\frac{u+1-u+1}{(u+1)^2}\right) = \frac{4(u-1)}{(u+1)^3}$

$\frac{du}{dx} = -2x^{-3}$

At $x = -1 \Rightarrow u = 0$

$\Rightarrow \frac{dz}{dx}\Big|_{x=-1} = \frac{dz}{du}\Big|_{x=-1} * \frac{du}{dx}\Big|_{x=-1} = \frac{dz}{du}\Big|_{u=0} * \frac{du}{dx}\Big|_{x=-1}$

$= \frac{-4}{1} * 2 = -8$