King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 7 Quiz I(A) (Term 163)

Name : ......KEY...................... ID #.................................... Serial #: ....

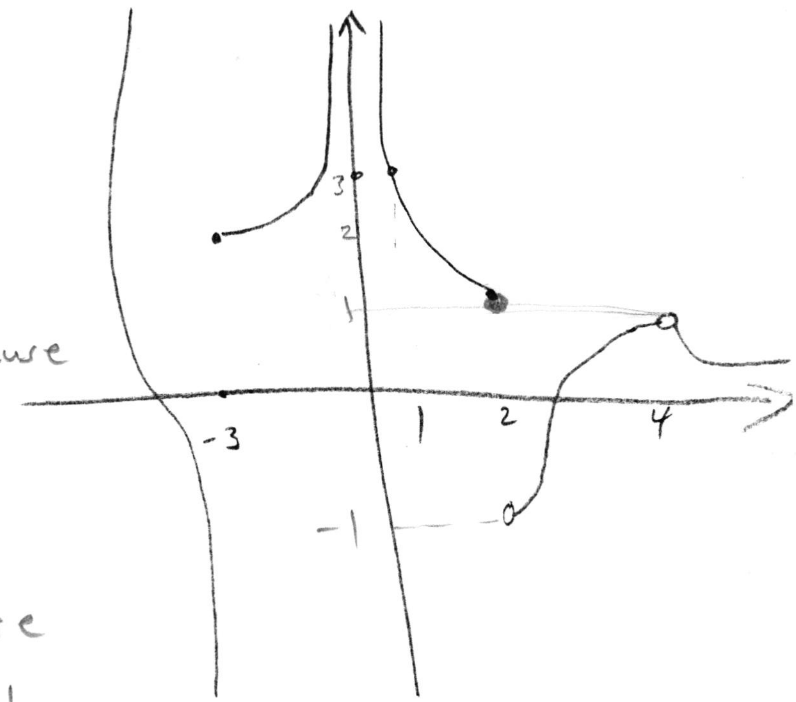
1. Evaluate the limit if it exists. If it does not exist, explain why? Use the symbols ∞ or $-\infty$ when needed

(i) $\lim_{x\to-3} f(x)$ DNE because

$\lim_{x\to-3^+} f(x) = 2$ but $\lim_{x\to-3^-} f(x) = -\infty$

(ii) $\lim_{x\to 0} f(x) = +\infty$ because

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = +\infty$



(iii) $\lim_{x\to 1} f(x) = f(1) = 3$

(iv) $\lim_{x\to 2} f(x)$ DNE because

$\lim_{x\to 2^-} f(x) = -1$ but

$\lim_{x\to 2^+} f(x) = 1$

(v) $\lim_{x\to 4} f(x) = 1$ because

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = 1$

(vi) $\lim_{x\to 4^-} (2f(x) + 3g(x))$ where $g(x) = [x-1]$ and $[x]$ is the greatest integer $\leq x$.

$= 2\lim_{x\to 4^-} f(x) + 3\lim_{x\to 4^-} g(x) = 2(1) + 3(2) = 8$

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2. Evaluate the limit or show that it does not exist:

(i) $\lim_{x\to 2^-} \dfrac{|x^2-6x+8|}{2x-4}$ $= \lim_{x\to 2^-} \dfrac{|(x-4)(x-2)|}{2(x-2)}$

$= \lim_{x\to 2^-} \dfrac{|x-4|\,|x-2|}{2(x-2)} = \lim_{x\to 2^-} \dfrac{(-(x-4))(-(\cancel{x-2}))}{2(\cancel{x-2})}$

$= \lim_{x\to 2^-} +\frac{1}{2}(x-4) = \frac{1}{2}(-2) = -1$

(ii) $\lim_{x\to \frac{\pi}{2}^+} [1-\cos x]$ where $[x]$ is the greatest integer $\leq x$

$= 1$

(iii) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x+2 & x \leq 1 \\ 2x-3 & x > 1 \end{cases}$

$\lim_{x\to 1^+} f(x) = 2-3 = -1$ but

$\Rightarrow \lim_{x\to 1} f(x)$ DNE

$\lim_{x\to 1^-} f(x) = 1+2 = 3$

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(iv) $\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1} \cdot \frac{\sqrt{6-x}+2}{\sqrt{6-x}+2} \cdot \frac{\sqrt{3-x}+1}{\sqrt{3-x}+1}$

$$= \lim_{x\to 2} \frac{(\cancel{6-x-4})(\sqrt{3-x}+1)}{(\cancel{3-x-1})(\sqrt{6-x}+2)} = \frac{1+1}{2+2} = \frac{1}{2}$$

(v) $\lim_{u\to -2} \frac{u^3+8}{u^2-4} = \lim_{u\to -2} \frac{(u+2)(u^2-u+4)}{(u+2)(u-2)}$

$$= \frac{4+4+4}{-4} = -3$$

(vi) $\lim_{x \to 0} x^2 \sin \frac{\pi}{x}$

We have $-1 \le \sin \frac{\pi}{x} \le 1$

$$\Rightarrow -x^2 \le x^2 \sin \frac{\pi}{x} \le x^2$$

as $x \to 0$: $-x^2 \to 0$

as $x \to 0$: $x^2 \to 0$

by the Squeeze theorem: $x^2 \sin \frac{\pi}{x} \to 0$

$\therefore \lim_{x \to 0} x^2 \sin \frac{\pi}{x} = 0$

3. Find all vertical asymptotes for the function

$$f(x) = \frac{(x-1)\ln(x)}{x^2-1}$$

possible V.A. are $x = 0$ & $x = \pm 1$

Domain: $x > 0$ & $x \neq \pm 1 \Rightarrow D_f: (0,1) \cup (1,\infty)$

$x = -1$ excluded

$x = 1$ $\quad \lim_{x \to 1^+} \frac{\cancel{(x-1)} \ln x}{\cancel{(x-1)}(x+1)} = \frac{0}{2} = 0 \Rightarrow$

$x = 1$ is not a V.A.

$x = 0$ $\quad \lim_{x \to 0^+} \frac{\ln x}{x+1} = -\infty \Rightarrow x = 0$ is a V.A.

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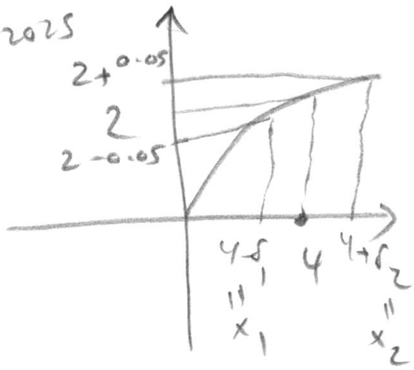
4. Use the graph of $y = \sqrt{x}$ to find a number $\delta > 0$ such that if $0 < |x-4| < \delta$, then $|\sqrt{x} - 2| < 0.05$.

$f(x) = \sqrt{x}$, $a = 4$, $L = 2$ & $\varepsilon = 0.05$

$\Rightarrow f(x_1) = 1.95$ & $f(x_2) = 2.05$

$\Rightarrow \sqrt{x_1} = 1.95$ & $\sqrt{x_2} = 2.05$

$\Rightarrow x_1 = (1.95)^2 = 3.8015$ & $x_2 = (2.05)^2 = 4.2025$



$\Rightarrow 4 - \delta_1 = 3.8015$ & $4 + \delta_2 = 4.2025$

$\Rightarrow \delta_1 = 0.1985$ $\Rightarrow \delta_2 = 0.2025$

$\Rightarrow \delta = \min\{\delta_1, \delta_2\} = 0.1985$

5. If $\displaystyle\lim_{x\to 0} \frac{\sqrt{ax+b} - 2}{x} = 1$, then find the values of a and b.

First, for the limit to be exist, we need $\sqrt{ax+b} - 2$ to be equal to 0 at $x = 0$. That means

$\sqrt{b} - 2 = 0 \Rightarrow b = 4$

$$\Rightarrow \lim_{x\to 0} \frac{\sqrt{ax+4} - 2}{x} \cdot \frac{\sqrt{ax+4} + 2}{\sqrt{ax+4} + 2} = \lim_{x\to 0} \frac{ax+4-4}{x(\sqrt{ax+4} + 2)}$$

$$= \frac{a}{4} = 1 \Rightarrow a = 4.$$