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Department of Mathematics & Statistics
Term 162 Quiz#4 (chap 5) STAT 460 (1)

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Q1. For the model $Y_t = 3 + Y_{t-1} + e_t - 0.1e_{t-1}$, answer the following:

a) Compute the mean $E[\nabla Y_t]$.

b) Compute $Var[\nabla Y_t]$. 0.1

Soln: a) $\nabla Y_t = Y_t - Y_{t-1} = 3 + e_t - \cancel{0.75}e_{t-1}$ so that $E(\nabla Y_t) = 3$

0.1

b) $Var[\nabla Y_t] = Var(3 + e_t - \cancel{0.75}e_{t-1})$ 1.01

$= Var(e_t - 0.75e_{t-1}) = (1 + \cancel{0.75}^2)\sigma_e^2 = \cancel{1.5625}\sigma_e^2$

0.1

Q2. Consider two models (A) $Y_t = 0.9Y_{t-1} + 0.09Y_{t-2} + e_t$ and (B) $Y_t = Y_{t-1} + e_t - 0.1e_{t-1}$,

a) Identify each model as a specific ARIMA model. (Hint: write what are the *p*, *d*, and *q* as well as the values of ϕ's and θ's.

b) Compare models A and B. That is, how are they different and how are they similar? (Hint: Compare the first 6 π-weights and ψ-weights)

Soln:

a) Since $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$ and $|\phi_2| < 1$ in model A, the process is a stationary AR(2) process. with $\phi_1 = 0.9$ and $\phi_2 = 0.09$.
Since $Y_t - Y_{t-1} = e_t - 0.1e_{t-1}$ in model B, this is an IMA(1,1) process with $\theta = 0.1$.

b) **Different:** (A) is stationary while the other (or B) is nonstationary.

Similar: Using Equations (4.3.21) on page 75, $\psi_1 - \phi_1\psi_0 = 0$, $\psi_j - \phi_1\psi_{j-1} - \phi_2\psi_{j-2} = 0 \quad j = 2, 3, \ldots$, we can calculate the ψ-weights for the AR(2) model. This could be done with a calculator:

$\psi_0 = 1$, $\psi_1 = \phi_1 = 0.9$, $\psi_2 = \phi_1\psi_1 + \phi_2\psi_0 = 0.9$,

$\psi_3 = \phi_1\psi_2 + \phi_2\psi_1 = 0.891$, $\psi_4 = \phi_1\psi_3 + \phi_2\psi_2 = 0.8829$,

$\psi_5 = \phi_1\psi_4 + \phi_2\psi_3 = 0.8748$, $\psi_6 = \phi_1\psi_5 + \phi_2\psi_4 = 0.866781$

From Equation (5.2.6), page 93, the ψ-weights for the IMA(1,1) model are $\psi_0 = 1$, $\psi_1 = 1 - \theta = 0.9$, $\psi_2 = 1 - 0.1 = 0.9$, $\psi_3 = 1 - 0.1 = 0.9$, $\psi_4 = 0.9$, $\psi_5 = 0.9$, $\psi_6 = 0.9$, … .

So the ψ-weights for the two models are very similar for many lags.

The π-weights for the IMA(1,1) model are obtained from Equation (4.5.5), $Y_t = \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \pi_3 Y_{t-3} + \ldots + e_t$ on page 80. We find that $\pi_k = (1 - \theta)\theta^{k-1}$ for $k = 1, 2, \ldots$. So $\pi_1 = (1 - 0.1) = 0.9$, $\pi_2 = (1 - 0.1)(0.1) = 0.09$, $\pi_3 = (1 - 0.1)(0.1)^2 = 0.009$, and so on.

The first two π-weights for the two models are identical and the remaining π-weights are nearly the same.

These two models would be essentially **impossible to distinguish** in practice.

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