King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT301 : Introduction to Probability Theory (162)
First Exam **Thursday May 11, 2017**

Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	12	
Two	14	
Three	10	
Four	10	
Five	10	
Six	14	
Seven	10	
Eight	9	
Nine	14	
Ten	17	
Total	120	

Question.1 (**5+7=12-Points**)

Two balls are chosen randomly from an urn containing 6 white, 3 black, and 1 green balls. We win \$5.0 for each black ball selected and we lose \$3.0 for each white ball selected. Let X be a random variable denotes to our wining.

(a) What are the possible values of the random variable X?

(b) Find the probability distribution of the random variable X.

Question .2 **(7+7=14-Points)**

A large lot of items is known to have a fraction of (p) defective $(0 < p < 1)$. Let X be a random variable represents the number of items to be inspected to obtain the second defective item.

(a) Find the probability distribution of X.

(b) Find the expected value of X

Question.3 **(6+2+2=10-Points)**

Suppose that the random variable X takes on one of the values $0, 1$ and 2. If for some constant c, $P(X = i) = cP(X = i - 1)$, $i = 1, 2$.

(a) Find the probability mass function of X.

(b) Find the expected value of X

(c) Find the cumulative distribution function of X

Question.4 **(10-Points)**

Let X be a binomial random variable with parameters (n, p). Show that $E\left\{\frac{1}{X+1}\right\} = \frac{1-(1-p)^{n+1}}{(n+1)p}$

Question 5. **(10-Points)**

Suppose that an airplane engine will operate during any flight with probability $1-p$, independently from engine to engine. Suppose that the airplane will make a successful flight if at least 50% of its engines remain operative. Find the values of p for which a 4-engine plane is preferable to a 2-engine plane.

Question 6. (**6+8=14-Points**)

(a) A certain type of agency employs 2 typists. The average number of errors per article is 4.2 when typed by the first typist and 3 when typed by the second. If your article will have twice chance to be typed by the first compared with the second one. What is the probability it will have no errors?

(b) An oil company conducts a geological study that an exploratory oil well should have 20% chance of striking oil.

(i) What is the probability that the first strike will comes on the third well drilled?

(ii) What is the probability that the third strike will comes on the seventh well drilled?

(iii) What is the mean and variance of the number of wells that must be drilled if the oil company wants to setup three producing wells?

Question 7. (**6+4=10-Points**)

Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. For a fixed number x_0, define the function $g(x) = \begin{cases} \frac{f(x)}{1-F(x_0)}, & x \geq x_0 \\ 0, & x < x_0 \end{cases}$

(a) Show that $g(x)$ is a pdf.

(b) Find the distribution function $G(x)$ of the pdf $g(x)$

Question 8. (**9-Points**)

A point is chosen at random on a line segment of length L. Find the probability that the ratio of the longer to the shorter segment is more than $\frac{7}{3}$.

Question 9. (**5+5+4=14-Points**)

The actual weight of a sugar bag follow a normal distribution with a mean of 204 grams and a standard deviation of 3 grams. I a bag weights less than 201 or more than 208 , it will be rejected.

(a) Find the probability that a bag will be rejected

(b) Given that the bag is rejected, what is the probability that it weights less than 198?

(c) what is the probability that out of five bags, three will be rejected?

Question 10. (**4+6+7=17-Points**)

(a) If 65% of the population of large community is in favor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain:

(i) At least 50 who are in favor of the proportion?

(ii) Exactly 70 in favor.?

(b) If X is uniformly distributed over $(0, 1)$, find the density of $Y = X^2$