HW 1:

Exercise 1: Let A and B be events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Show that
$$\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3},$$
and give examples to show that both extremes are possible.

Exercise 2: Let A_r, $r \geq 1$, be events such that $P(A_r) = 1$ for all r. Show that $P\left(\bigcap_{r=1}^{\infty} A_r\right) = 1$.

Exercise 3: Prove that $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$ whenever $P(A)P(B) \neq 0$. Show that, if $P(A|B) > P(A)$, then $P(B|A) > P(B)$.

Exercise 4: Let A and B be independent events; show that A^c, B are independent, and deduce that A^c, B^c are independent.

Exercise 5: Prove Bayes's formula: If $A_1, A_2, \ldots, A_n$ is a partition of Ω, then:
$$P(A_j|B) = \frac{P(B|A_j)\,P(A_j)}{\sum_{i=1}^{n} P(B|A_i)\,P(A_i)}.$$