King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 601
Exam I– 2016–2017 (162)
Tuesday, April 25, 2017

Allowed Time: 90 minutes

Instructor: **Dr. Boubaker Smii**

Name: _____

ID #: _____

Section #: _____ Serial Number: _____

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification !

Question #	Grade	Maximum Points
1		**10**
2		**10**
3		**10**
4		**20**
Total:		**50**

Exercise 1:

Let ψ be a characteristic function of a given random variable X.

1- Prove that

$$Re\Big(1-\psi(t)\Big) \geq \frac{1}{4}\, Re\Big(1-\psi(2t)\Big),\ t \in \mathbb{R}. \tag{a}$$

2- Deduce from equation (a) that

$$1- \mid \psi(2t) \mid \leq 8\left(1- \mid \psi(t) \mid \right),\ t \in \mathbb{R}. \tag{b}$$

Exercise 2:

1- Let $\{N(t),\, t \geq 0\}$ be a Poisson process with rate λ.

i)- Find $\mathbb{E}(N(t))$.

ii)- Assume that $\lambda = 4$. Compute: $\mathbb{E}(3N(2) - 5N(4))$.

2- Let X and Y be two random variables with a Poisson distributions with parameters λ_1 and λ_2 respectively. If X and Y are independent, find the distribution of the random variable $X + Y$.

Exercise 3:

Let $t_0^n < t_1^n < \cdots < t_n^n = T$, where $t_i^n = i\frac{T}{n}$, be a partition of the interval $[0, T]$ into n equal parts. We denote by

$$\Delta_i^n B = B(t_{i+1}^n) - B(t_i^n) \tag{c}$$

the corresponding increments of the Brownian motion $B(t)$.

Show that

$$\lim_{n \longrightarrow +\infty} \sum_{i=0}^{n-1} (\Delta_i^n B)^2 = T, \quad in \ \ L^2. \tag{d}$$

Exercise 4: I- Consider the standard Brownian motion $\{B_t,\, t \geq 0\}$.

a)- Show whether or not $V_t = \sqrt{t}\, B_t$ is a standard Brownian motion.

b)- Find $\mathbb{E}(\mid B_t - B_s \mid^2)$.

c)- Given that $\displaystyle\int_{\mathbb{R}} e^{\frac{-(x - i\,\lambda\, t)^2}{2t}}\, dx = \sqrt{2\,\pi\, t}$, compute the characteristic function of B_t.

d)- Deduce from c) $\mathbb{E}(B_t^4)$.

II- Give two major differences between the Riemann and Itô integrals.