King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Exam I - Math 568 (162)   Time: 2 hours

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Name: ID #
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Please show all work. No credit for a result without work

Problem 1	/8
Problem 2	/10
Problem 3	/11
Problem 4	/6
Total	/35

Problem # 1. (8 marks) Find the function $u(x, y, z)$ that solves the problem

$$u_x + xu_y - u_z = u,$$
$$u(x, y, 0) = x + y$$

Problem # 2. (10 marks) Use the characteristic method to find the solution of

$$u_x^2 + yu_y = u, \qquad u(x,1) = 1 + \frac{x^2}{4}$$

Problem # 3. (11 marks) Let

$$u_{xx} + 3u_{xy} - 4u_{yy} + u_x + 4u_y = 0$$

a. By an appropriate change of variable, reduce it to the canonical form

$$w_\eta - 5w_{\xi\eta} = 0$$

b. Find the general solution $u(x, y)$.

Problem # 4. (6 marks) Given the coupled problem

$$(S)\quad \begin{cases} u_t - v_x = 0 \\ v_t - 4u_x = 0 \\ u(x,0) = x, \;\; v(x,0) = 1 \end{cases}$$

a. verify that (S) can be written as

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} u_x \\ v_x \end{pmatrix}$$

b. find the eigenvalues λ_1 and λ_2 of $A = \begin{pmatrix} 0 & 1 \\ 4 & 0 \end{pmatrix}$ and a matrix P such that

$$P^{-1}AP = D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

c. Let $\begin{pmatrix} \phi \\ \psi \end{pmatrix} = P^{-1} \begin{pmatrix} u \\ v \end{pmatrix}$. Show that we have

$$\begin{pmatrix} \phi_t \\ \psi_t \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \phi_x \\ \psi_x \end{pmatrix} \tag{0.1}$$

d. Solve the decoupled system (0.1) and find the solution of (S)