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King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 513 Midterm Exam, Term 162
Duration: 120 minutes

Name: ……..

1) Find the transfer functions, impulse response, and step response for $y'' - y' - 6y = f(t)$. Assume that all of the necessary initial conditions are zero.

2) Find the Fourier series for

$$f(x) = \begin{cases} 2, & 0 < x < 1, \\ 0, & 1 < x < 4. \end{cases}$$

To what value does this series converge to at $x = -3$?

3) Use the definition to find the convolution $e^{-t}H(t) * e^{-t}H(t)$ related to the Fourier transform.

4) Derive the Fourier transform for $f(t) = e^{-|t|}$.

5) Find a particular solution for $y'' - 4y = \delta(t)$.

6) Find $\mathcal{L}\{f(t)\}$ of

$$f(t) = \begin{cases} 1, & 0 \le t \le 2, \\ -1, & t > 2. \end{cases}$$

7) Show that $\{\cos nx\}_{n=0}^{\infty}$ is an orthogonal set on $[0, \pi]$. Find the norm of each function.

8) Find the first three non-vanishing coefficients in the Legendre polynomial expansion of

$$f(x) = \begin{cases} x, & -1 < x < 0, \\ 0, & 0 < x < 1. \end{cases}$$

Q	1	2	3	4	5	6	7	8	Total
Max	12	14	12	12	12	12	12	14	100
Points									

1. The Fourier transform of $f(t)=\sin(t)\,H(t)$ is $F(\omega)=\dfrac{1}{1-\omega^2}+\dfrac{\pi i}{2}[\delta(\omega+1)-\delta(\omega-1)]$. Find the Fourier transform of

$$f(t)=\sin(t-1)\,H(t-1).$$

2. Find $\mathcal{L}^{-1}\left\{\dfrac{s^2}{s^2+1}\right\}$.

3. Use the Laplace transform to solve: $y'-2y=t$, $y(0)=2$.

4. Let $f(t)=t-1$ and $g(t)=e^t$. Find the convolution related to the Laplace transform.