King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 430
Final Exam – 2016–2017 (162)
Allowed Time: 180 minutes

Name: ______________________________________

ID #: ________________________

Section #: __________ Serial Number: ______________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**

Problems:

Question #	Grade	Maximum Points
1		9
2		7
3		10
4		12
5		10
6		10
7		20
8		18
Total:		96

Q:1 (2 + 4 + 3 points) (a) Find all solutions of the equation

$$z^8 - 2z^4 + 1 = 0.$$

(b) Sketch the set of points in the complex plane satisfying the inequality $Im(\frac{1}{z}) > 1$ ($z \neq 0$).
(c) Find the value of i^{2i}.

Q:2 (3 + 4 points) (a) Find all roots of $\cosh(z) = \dfrac{1}{2}$.
(b) Show that $|\sin(z)|^2 = \sin^2 x + \sinh^2 y$.

Q:3 (10 points) Evaluate $\int_C \text{Im(z - i) dz}$, where C is the polygonal path consisting of the circular arc along $|z| = 1$ from $z = 1$ to $z = i$ and the line segment from from $z = i$ to $z = -1$.

Q:4 (2 + 6 + 4 points) (a) State the maximum modulus principle.

(b) Let $f(z) = e^z$ and R be the rectangular region $0 \leq x \leq 1$ and $0 \leq y \leq \pi$. Find the points in R, where $v(x, y) = Im\ f(z) = \mathrm{e^x}\ \sin(\mathrm{y})$ reaches its maximum and minimum values.

(c) Use Cauchy's integral formula to evaluate

$$\int_{|z|=1} \frac{\cos(\mathrm{z})}{z(z^2+9)}\, dz,$$

Q:5 (10 points) Show that when $0 < |z-1| < 2$,

$$\frac{z}{(z-1)(z-3)} = -3\sum_{n=0}^{\infty}\frac{(z-1)^n}{2^{n+2}} - \frac{1}{2(z+1)}.$$

Q:6 (10 points) Use Cauchy's residue theorem to evaluate

$$\int_{|z|=2} \frac{\mathrm{dz}}{\sinh(2\ \mathrm{z})}.$$

Q:7 (8 + 8 + 4 points) (a) Let $|F(z)| \leq \frac{M}{R^k}$ for $z = Re^{i\theta}$, where $k > 0$ and M are constants. Prove that

$$lim_{R\to\infty} \int_C e^{imz} F(z) dz = 0,$$

where C is the semicircular arc of radious R $(0 \leq \theta \leq \pi)$ and m ia constant.

(b) Evaluate $\int_0^\infty \dfrac{dx}{(x^2 + 1)^2}$.

(c) Use Rouche's theorem to show that the polynomial $z^6 + 4z^2 - 1$ has exactly two zeros in the disk $|z| = 1$.

Cont.

Q:8 (6 + 2 + 10 points) (a) Determine where the complex mapping $w = z + \frac{1}{z}$ is conformal. Show that the mapping $w = z + \frac{1}{z}$ maps $|z| = \rho$ $(\rho \neq 1)$ onto the ellipse.
(b) Find the scale factor of $w = z^2$ at the point $2 + i$.
(c) Show that the transformation $w = sin(z)$ is a one to one conformal mapping of the vertical strip $|x| < \frac{\pi}{2}$ onto the w-plane slit along the rays $u \leq -1, v = 0$ and $u \geq 1, v = 0$.