King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430 Exam 03
The Second Semester of 2016-2017 (162)
Time Allowed: 90 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		11
2		6
3		8
4		16
5		10
Total		51

Q1: (3 + 8 points) (a) State and prove Gauss's mean value theorem..

(b) Use Cauchy's integral formula(s) to evaluate

$$\int_C \frac{z^3+3}{z(z-i)^2}dz,$$

where C is the contour shown in the fig.

Q2: (6 points) Let C be the unit circle $z = e^{i\,\theta}$ $(-\pi \le \theta \le \pi)$. First show that for any real constant a, $\int_C \frac{e^{az}}{z}dz = 2\pi i$. Then write this integral in terms of θ to derive the integral formula

$$\int_0^\pi \mathrm{e}^{\mathrm{a}\ \cos(\theta)} \quad \cos(\mathrm{a}\ \sin\theta)\mathrm{d}\theta \ = \ \pi.$$

Q3: (8 points) State and prove the fundamental theorem of algebra for any polynomial $P(z)$ of degree $n(n \ge 1)$.

Q4: (4 + 4 + 4 + 4 points) Let $f(z) = (z^2 - 3z + 2)^{-1}$. Find the Laurent series for $f(z)$ valid for

(a) $1 < |z| < 2$ (b) $|z| < 1$ (c) $|z| > 2$ (d) $0 < |z - 1| < 1$

Q5: (5 + 5 points) (a) Expand $f(z) = \frac{1}{1-z}$ in a Taylor series with center $z_0 = 2i$ and find the circle of convergence.

(b) Evaluate

$$\int_C \frac{1}{z^2\ \sinh(\mathrm{z})}dz,$$

where C is the positively oriented circle $|z| = 1$ and

$$\frac{1}{\sinh(\mathrm{z})} = \frac{1}{z} - \frac{1}{6}.z + \frac{7}{360}z^3 + ... \qquad (0 < |z| < \pi).$$