King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430   Major Exam II
The Second Semester of 2016-2017 (162)
<u>**Time Allowed: 90 Minutes**</u>

Name: _____   ID#: _____

Section: _____   Serial #: _____

- Mobiles and calculators are not allowed in this exam.

- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		18
2		18
3		15
Total		51

Q1: $(3 + 6 + 9$ points) (a) Define harmonic function for a real valued function of two variables.

(b) Verify that the function $u(x, y) = x\, y^3 - x^3\, y$ is harmonic in the entire complex plane. Find a harmonic conjugate of $u(x, y)$.

(c) State and prove **reflection principle**.

Q2: $(6 + 6 + 6$ points) (a) (i) Find all values of $(1 + i)^{(1-i)}$.

(ii) Solve the equation $\cos z = \sqrt{2}$.

(b) Find all roots of the equation $\cos(z) = \cosh 2$ by equating the real and imaginary parts of $\cos(z)$ and $\cosh 2$.

(c) Find all roots of the equation $\sinh(z) = i$.

Q3: $(2 + 5 + 8$ points) (a) Show that if $w(t) = u(t) + i\, v(t)$ is continous on $a \le t \le b$, then

$$\int_a^b w(t)\, dt = \int_\alpha^\beta w[\phi(\tau)]\, \phi'(\tau)d\tau,$$

where $\phi(\tau) = t, \alpha \le \tau \le \beta$.

(b) Find the **upper bound** of $\oint_C \dfrac{z+4}{z^3 - 1}dz$, where C is the circle $|z|$ from $z = 2$ to $z = 2i$

(c) State the **Cauchy-Goursat theorem** and use it to evalute $\oint_C \dfrac{z+1}{z^2 + 2z - 3}dz$, where C is the circle $|z - 2| = 2$.