

**King Fahd University of Petroleum & Minerals**  
**Department of Mathematics & Statistics**  
**Math 430 Major Exam I**  
**The Second Semester of 2016-2017 (162)**

**Time Allowed: 90 Minutes**

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Name: \_\_\_\_\_ ID#: \_\_\_\_\_

Section/Instructor: \_\_\_\_\_ Serial #: \_\_\_\_\_

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- Mobiles and calculators are not allowed in this exam.
  - Provide all necessary steps required in the solution.
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Question #	Marks	Maximum Marks
1		15
2		12
3		11
4		12
Total		50

**1** (15 points) (a) Verify that  $\sqrt{2} |z| \geq |Re z| + |Im z|$ .

(b) Sketch the set of points determined by  $|z - i| = \text{Im } z + 1$ .

(c) Express  $\frac{1+i}{\sqrt{3}-i}$  in terms of rectangular and polar forms.

**(2)** (12 points) (a) Show that the all values of  $f(z) = \frac{z}{1-z^2}$  lies on the y-axis.

(b) If  $z$  is a complex number satisfying  $|z|^2 - |z| - 2 < 0$ , then find the value of  $|z^2 + z \sin \theta|$  for all values of  $\theta$ .

**(3)** (11 points) (a) Find the roots of  $(-16)^{\frac{1}{4}}$  and locate them graphically.

(b) Define " Domain and open set " in the complex plane.

(c) Is the set  $0 < |z - 2| < 3$  a domain and open set? Give reason.

**(4)** (12 points) (a) Use  $\epsilon - \delta$  definition to show that  $\lim_{z \rightarrow 1} \frac{i \bar{z}}{2} = \frac{i}{2}$

(b) State the Cauchy-Riemann equations for a differentiable function  $f(z)$  at point  $z_0$ .

(c) Let  $f(z) = \frac{1}{z^2}$  ( $z \neq 0$ ). Use the Cauchy-Riemann equations in polar coordinates to find  $f'(z)$ .