King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester – 162

Exam III **Dr. M. Z. Abu-Sbeih** **May 6, 2017**

Student No.: __________. Name: ___________________________________

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (18 points): Prove or disprove:

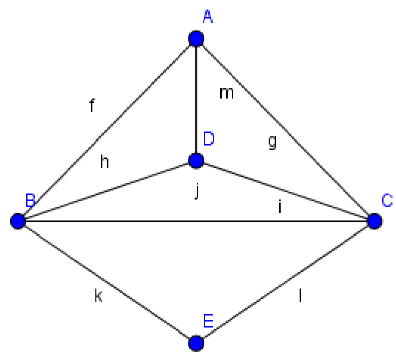
(a) A transitive tournament is acyclic.

(b) Let v be a vertex of maximum out-degree in a tournament T of order n. Show that $\vec{d}(v, u) \leq 2$ for all vertices $u \in V(G)$.

Problem 2 (27 points):

(a) Find the crossing number of the complete graph K_{10} and the complete bipartite graph $K_{7,10}$.

(b) Find the automorphism group of the graph G in the figure.



(c) Find the number of distinct labeling of the graph G in part (b) from a set of n labels.

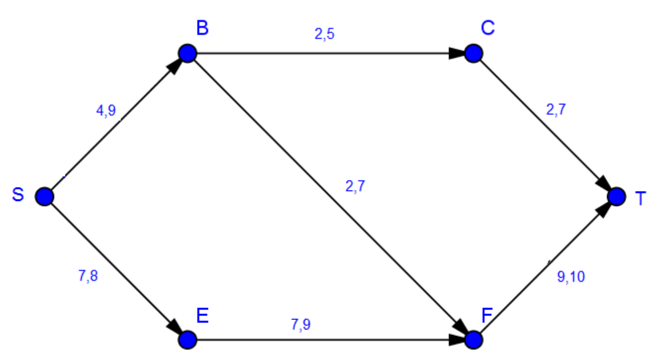
Problem 3 (20 points): Consider the network in the following figure, where the first number on an edge e indicates the flow $f(e)$ and the second number indicates the capacity $c(e)$ of the edge.

Find:

(a) the $val(f) =$

(b) the capacity of a minimum cut.

(c) Is the given flow maximal flow? Why?



Problem 4 (36 points):

(a) Prove that the size of a maximal outerplanar graph G of order $n \geq 3$ is equal to $2n - 3$. Find the number of faces of G.

(b) Find all connected regular plane graphs with number of faces equal to the order of the graph.

(c) State Euler's formula (identity) for connected planar graphs.
If a connected planar graph has order n and size m and no triangles, prove that $\leq 2n - 4$.

(d) State Kuratowski's Theorem.
Which of the following graphs is planar? $K_6 - e$, $K_{4,3}$, Q_4 , the Petersen graph.
Explain why.