King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester – 162

Exam II **Dr. M. Z. Abu-Sbeih** **April 18, 2017**

Student No.: _____. Name: _____

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (18 points):

(A) Consider the graph $G = K_5 - e$; (the complete graph on 5 vertices minus one edge). Find

(i) $\kappa(G) =$

(ii) $\lambda(G) =$

(iii) $\alpha(G) =$

(iv) $t(G) =$

(B) Write the Statement of each of the following Theorems in detail.

(i) Menger's Theorem

(ii) Ore's Theorem.

Problem 2 (22 points): Consider the graph $= K_{1,3}$.

(A) Sketch the graph of each of the following:

a. $S(G)$
b. $L(G)$
c. G^2
d. $T(G)$

(B) Which of the 4 graphs in (A) is Eulerian?

(C) Which of the 4 graphs in (A) is Hamiltonian?

Problem 3 (28 points): *Either prove or disprove each of the following statements. If a statement is true proof it, and if it is false, give a counter example.*

1) If D is an Eulerian digraph then $L(D)$ is Eulerian.

2) If D is a digraph of order n such that $d(v) \geq n$ for every vertex v of D, then D is Hamiltonian.

3) If G is Hamiltonian graph, then $L(G)$ is Hamiltonian-connected.

4) If a graph G is pancyclic then it is panconnected.

Problem 4 (32 points): Prove each of the following

(a) If G is an Eulerian graph of odd order then G has 3 vertices of the same degree.

(b) Let G be a k-connected graph of diameter k, where $k \geq 2$, prove that G contains $k+1$ distinct vertices $v_1, v_2, \ldots, v_k$ and k internally disjoint $v - v_i$ paths P_i $(1 \leq i \leq k)$ such that P_i has length i.

(c) Show that if G is a graph of order at least 2 for which $\kappa(G) \geq \alpha(G) - 1$, then G has a Hamiltonian path.

(d) If a graph G is Hamiltonian-connected of order 4 or more, then it is 3 connected.