**King Fahd University of Petroleum & Minerals**

**Department of Mathematics and Statistics**

**Math 421: Introduction to Topology**

**Final Exam, Fall Semester 162 (180 minutes)**

**Jawad Abuhlail**

==================================================

**Q1. (30 points)** Show that
(a) compactness is a strong topological property.
(b) every compact Hausdorff space is regular.
(c) every metric space is normal.

**Q2. (10 points)** State whether true or false:
(1) $\mathbb{R}^2_{\mathcal{U}}$ is compact
(2) $\mathbb{R}^2_{\mathcal{U}}$ is normal
(3) $\mathbb{R}^2_{\mathcal{L}}$ is regular
(4) $\mathbb{R}_{\mathcal{L}}$ is metrizable
(5) $\mathbb{R}_{\mathcal{CF}}$ is compact
(6) $\mathbb{R}^n_{\mathcal{U}}$ is path-connected
(7) $\mathbb{R}^3_{\mathcal{U}} \cong \mathbb{R}^2_{\mathcal{U}}$
(8) $\mathbb{R}_{\mathcal{RR}} \cong \mathbb{R}_{\mathcal{RL}}$
(9) $\mathbb{Q}_{\mathcal{U}}$ is connected
(10) $\mathbb{R}_{\mathcal{RR}}$ is compact.

**Q3. (10 points)** Show that if $f : (X, \tau) \to (Y, \upsilon)$ is continuous, with $Y$ compact and Hausdorff, then

$$G_f := \{(x, f(x)) \in X \times Y : x \in X\}$$

is closed in $X_\tau \times Y_\upsilon$.

**Q4. (10 points)** Show that a topological space $(X, \tau)$ is connected if and only if $X$ is *not* the union of two non-empty separated sets.

**Q5. (10 points)** Show that every compact subset of $\mathbb{R}_{\mathcal{L}}$ is countable.

**Q6. (30 points)** Prove or disprove:

**(a)** $\mathbb{R}_{\mathcal{RR}}$ is metrizable.

**(b)** If $(X, \tau)$, $(Y, \upsilon)$ are topological spaces, $\{A_\lambda\}_{\lambda \in \Lambda}$ is a collection of closed subsets of $X$ such that $X = \bigcup_{\lambda \in \Lambda} A_\lambda$ and $f : X_\tau \to Y_\upsilon$ is a function such that $f : A_\lambda \to Y_\upsilon$ is continuous for each $\lambda \in \Lambda$, then $f : X_\tau \to Y_\upsilon$ is continuous.

**(c)** The unit circle (with the usual topology) is $T_4$.

# GOOD LUCK