King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 421: Introduction to Topology

First Exam, Fall Semester 162 (120 minutes)

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Q1. (10 points) Let (X, τ) be a topological space and $A \subseteq X$. Show that

$$Int(A) = \bigcup_{V \subseteq A,\ V \in \tau} V.$$

Q2. **(15 points)** Show that
(a) $\mathbb{R}_{\mathcal{L}} \cong \mathbb{R}_{\mathcal{R}}$.
(b) $(0, \infty) \cong (0, 1)$ (as topological subspaces of $\mathbb{R}_{\mathcal{U}}$).

Q3. (15 points) Consider

$$g : \mathbb{R} \to \mathbb{R},\ g(x) = \begin{cases} x+1, & x > 1 \\ \\ x, & x \leq 1 \end{cases}$$

Is g $\mathcal{L} - \mathcal{U}$ continuous? $\mathcal{L} - \mathcal{L}$ continuous? $\mathcal{U} - \mathcal{L}$ continuous? (Justify your answer)

Q4. (30 points) Let (X, τ), (X', τ') be topological spaces.

(a) Let $f : X \to X'$ be a function. Show that if $X = \bigcup_{\lambda \in \Lambda} A_\lambda$ with $A_\lambda \in \tau$ and $f_{|A_\lambda} : A_\lambda \to X'$ is continuous for each $\lambda \in \Lambda$, then f is continuous.

(b) Let $X = A \cup B$, where A, B are *closed* subsets of A. Let $f : A \to X'$ and $g : B \to X'$ be continuous, such that $f(x) = g(x)$ for all $x \in A \cap B$. Show that

$$h : X \to X',\ h(x) = \begin{cases} f(x), & x \in A \\ \\ g(x), & x \in B \end{cases}$$

is continuous.

Q5. (30 points) Prove or disprove:

(a) If $f : \mathbb{R} \to \mathbb{R}$ is $\mathcal{U} - \mathcal{U}$ continuous, then f is $\mathcal{L} - \mathcal{U}$ continuous.

(b) If $\mathbb{R} \to \mathbb{R}$ is $\mathcal{U} - \mathcal{U}$ continuous, then f is a *closed* (the image of any closed set is closed).

(c) $Int(A \cup B) = Int(A) \cup Int(B)$ for any two subsets A, B of a topological space (X, τ).

GOOD LUCK