KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 345 : EXAM 2, T 162, MAY 03, 2017

Time : 19 : 00 * * * 21 : 00

Name : ....

ID : ....

Exercise 1. Use Lagrange's theorem to prove the Fermat's little theorem : If p is a prime number and a is an integer such that p does not divide a, then

$$a^{p-1} \equiv 1 \pmod{p}.$$

Exercise 2. Show that $\mathbb{Q}/\mathbb{Z}$ is an infinite group where every element is of finite order.

Exercise 3. Let p be a prime number. Determine the number of elements of order p in $\mathbb{Z}_{p^2} \oplus \mathbb{Z}_{p^2}$.

Exercise 4. Is a subgroup of a direct product of groups $G_1 \oplus G_2$ a direct product of subgroups $H_1 \oplus H_2$, where $H_i - -G_i$?

Exercise 5. Determine the number of cyclic subgroups of order 6 in $\mathbb{Z}_{36} \oplus \mathbb{Z}_9$.

Exercise 6. Consider the group $(\mathbb{R}^*, \times)$

(1) Find all subgroups H of $\mathbb{R}^*$ with index 2.

(2) Show that $\mathbb{R}^*/H \simeq \mathbb{Z}_2$.

(3) Let $K = \{2^n : n \in \mathbb{Z}\}$. Show that K is a cyclic subgroup of $\mathbb{R}^*$. Is $\mathbb{R}^*/K$ cyclic ?

Exercise 7. A group G is called metacyclic if it contains a normal cyclic subgroup H such that G/H is cyclic.

(1) Show that every cyclic group is metacyclic.

(2) Show that S_3 is a metacyclic noncyclic group.

(3) Show that in a metacyclic group, every subgroup is metacyclic.