MATH 280-01 (162)
HW # 6 and 7
Due on 9/5/2017

Q1. Let A be an $n \times n$ matrix such that $A^T A = I$. What is the rank of A? (Justify your answer)

Q2. Verify that the following is a linear operator on $\mathbb{R}^3$ and find a basis for its kernel:

$$L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3x - y + z \\ y + 2z \\ 3y + 6z \end{bmatrix}$$

Q3. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear operator and suppose that

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad \text{and} \quad T \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

Find the standard matrix representation of T.

Q4. Let $V = [1, x, x^2]$ and $U = [1, 1 + x, 1 + x + x^2]$ be two ordered bases for P_3.

(i) Find the transition matrix from V to U.

(ii) Use the matrix obtained in (i) above to find the coordinates of $P(x) = 2 + 5x - x^2$.

Q5. If A and B are similar $n \times n$ matrices, show that $(A^2 - 2A + I)$ and $(B^2 - 2B + I)$ are similar.

Q6. Let

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 & 1 & 4 \\ 2 & 4 & 5 & 5 & 4 & 9 \\ 3 & 6 & 7 & 8 & 5 & 9 \end{bmatrix}$$

(i) Find a basis for the column space of A.

(ii) Find a basis for the Null space of A.

(ii) What is the rank of A ?

Q7. Let $\mathbf{x} = (2, -5, 4)^T$ and $\mathbf{y} = (1, 2, -1)^T$. Find the vector projection $\mathbf{p}$ of $\mathbf{x}$ onto $\mathbf{y}$ and verify that $\mathbf{p}$ and $\mathbf{x} - \mathbf{p}$ are orthogonal.