

King Fahd University of Petroleum & Minerals  
Department of Mathematics and Statistics  
MATH 280-01(Term 162)  
Exam I  
15 March 2017

NAME: .....

ID #: .....

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Question	points	Score
1	12	
2	12	
3	12	
4	14	
5	10	
6	12	
7	12	
8	16	
Total	100	

Q1. Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Find  $A^{-1}$  and use it to solve the system

$$\begin{aligned} x_1 + 3x_3 &= -5 \\ x_1 + x_2 + 3x_3 &= 2 \\ x_2 + x_3 &= 5 \end{aligned}$$

Q2. Let

$$A = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & -3 \end{bmatrix}$$

Find the  $LU$  factorization of  $A$  and use it to find  $\det(A)$

Q3. Let  $A$  and  $B$  be  $3 \times 3$  matrices with  $\det(A) = x$  and  $\det(B) = y$ . Let  $E$  be a  $3 \times 3$  elementary matrix of type I. Use properties of determinants to compute (Justify your steps):

(i)  $\det(5A)$

(ii)  $\det(A^{-1}B)$

(iii)  $\det(EAB^T)$

Q4. Let  $S$  be a subset of  $\mathbb{R}^3$  such that

$$S = \{(x, y, z)^T \mid 2x - y + z = 0\}$$

Show that  $S$  is a subspace of  $\mathbb{R}^3$

Q5. Let  $A$  and  $B$  be  $n \times n$  symmetric matrices. Show that  $G = AB + BA$  is symmetric.

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Q6. Let  $A$  be a  $3 \times 3$  skew symmetric matrix ( $A^T = -A$ ). Show that  $A$  is singular.

Q7. Let  $A$  be a nonsingular matrix. Define the adjoint of  $A$  ( $adj A$ ). Show how to use it to find  $A^{-1}$

Q8. Show that if  $A$  is nonsingular then  $\text{adj}A$  is nonsingular and

$$(\text{adj}A)^{-1} = \det(A^{-1})A = \text{adj}A^{-1}$$