KFUPM/ Department of Mathematics & Statistics/ 162/ MATH 260-04/ Quiz 2

Name: **ID #:** **Serial #:**

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1. Find the dimension of the space of all vectors in $\mathbb{R}^4$ of the form (a, b, c, d) such that $2a + b = 3c$.

Solution. This space consists of all solutions of the homogeneous system $2a + b - 3c = 0$.
Coefficient matrix has echelon form $\begin{bmatrix} 2 & 1 & -3 & 0 \end{bmatrix}$, so the system has one leading variable and 3 free variables. Therefore the dimension of the space of solutions is 3.

2. Are the functions $f(x) = 2\cos x - 3\sin x$, $g(x) = 3\cos x + 2\sin x$ linearly dependent on $(-\infty, \infty)$? Justify.

Solution. Use the Wronskian: $W(f(x), g(x)) = \begin{vmatrix} 2\cos x - 3\sin x & 3\cos x + 2\sin x \\ -3\cos x - 2\sin x & 2\cos x - 3\sin x \end{vmatrix}$, so that $W(f(0), g(0)) = \begin{vmatrix} 2 & 3 \\ -3 & 2 \end{vmatrix} = 13 \neq 0$.
Hence the two functions are linearly independent on $(-\infty, \infty)$