KFUPM/ Department of Mathematics & Statistics/162/MATH 232/Quiz 2

Name: ID#: Serial #:

1. Prove that if n is a positive integer, then n^2+2 is not divisible by 4.

Solution. (Proof by cases) If n is a positive integer, then either it is odd or even.

- If n is odd, then $n = 2k+1$ for some integer k and hence $n^2+2 = (2k+1)^2 + 2 = 4(k^2+k)+3$. Since k^2+k is an integer, we obtain that n^2+2 has remainder 3 when divided by 4 and so cannot be divisible by 4.
- If n is even, then $n = 2k$ for some integer k and hence $n^2+2 = 4k^2+2$. Since k^2 is an integer, we obtain that n^2+2 has remainder 2 when divided by 4 and so cannot be divisible by 4.

  [A proof by cases can also be given using congruences.]

2. (a) Use the Euclidean algorithm to find $\gcd(630, 100)$.

(b) Find integers x, y such that $\gcd(630, 100) = 630x + 100y$.

Solution. (a) We have $630 = 6\times 100+30, \quad 100 = 3\times 30+10, \quad 30 = 3\times 10$. Hence $\gcd(630, 100) = 10$.

(b) We have $10 = 100 - 3\times 30 = 100 - 3\times(630 - 6\times 100) = 630\times(-3) + 100\times 19$. We can therefore take $x = -3$ and $y = 19$.