KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

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Exercise 1.

(1) Find the G.S of the system $X' = AX$, where $A = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$

(2) Solve the (IVP) : $X' = AX$, $X(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Exercise 2. Given that the characteristic polynomial of the matrix
$$A = \begin{pmatrix} 3 & 4 & -10 \\ 2 & 1 & -2 \\ 2 & 2 & -5 \end{pmatrix}$$
is $\det(\lambda I - A) = (\lambda - 1)(\lambda + 1)^2$, find the General Solution of the system of linear DEs : $X' = AX$.

Exercise 3. Given that the characteristic polynomial of the matrix
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & -1 \\ 0 & 2 & 2 \end{pmatrix}$$
is $\det(\lambda I - A) = (\lambda - 2)^3$, find the General Solution of the system of linear DEs :
$X' = AX$.

Exercise 4. Find a particular solution of the system
$$X' = A(t)X + t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix},$$
given that $M(t) = t \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}$ is a fundamental matrix for the complementary system.

Exercise 5. Let A be a square matrix.

(1) Show that if $A^2 = I$, then
$$e^{tA} = (\cosh t)I + (\sinh t)A.$$

(2) Use (1) to solve the system

$$X'(t) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} X(t).$$