MATH 202.5 (Term 162)

Quiz 6 (Sects. 8.2 & 8.3) Duration: 30min

Name: **ID number:**

1.)(4pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 6 & -1 \\ 1 & 4 \end{pmatrix} X.$

2.)(3pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} X.$

3.)(3pts)Solve the system $X' = AX + \begin{pmatrix} 3 \\ e^t \end{pmatrix}$**, given that** $\Phi(t) = \begin{pmatrix} e^t & -2e^{-2t} \\ e^t & e^{-2t} \end{pmatrix}$ **is a fundamental matrix of** $X' = AX$.

1a) $\begin{vmatrix} 6-\lambda & -1 \\ 1 & 4-\lambda \end{vmatrix} = 0$, $(6-\lambda)(4-\lambda)+1=0$

$\lambda^2-10\lambda+25=0$, $\lambda=5;5$

$(A-5I)K=0$

$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $x-y=0$ $K\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$

$X_2 = (tK+P)e^{5t}$, $(A-5I)P=K$

$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & -1 & 1 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right)$ $x-y=1$ $P\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$X_2 = \left[t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] e^{5t} = \begin{pmatrix} t+1 \\ t \end{pmatrix} e^{5t}$

$\boxed{X = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t} + c_2 \begin{pmatrix} t+1 \\ t \end{pmatrix} e^{5t}}$

2) $\begin{vmatrix} 3-\lambda & 4 \\ -4 & 3-\lambda \end{vmatrix} = 0$, $(3-\lambda)^2+16=0$

$\lambda = 3 \pm 4i$

$(A-(3+4i)I)K=0$

$\left(\begin{array}{cc|c} 3-3-4i & 4 & 0 \\ -4 & 3-3-4i & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} -4i & 4 & 0 \\ -4 & -4i & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} 1 & i & 0 \\ -i & 1 & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right)$ $x+iy=0$ $K\begin{pmatrix} i \\ -1 \end{pmatrix}$

$K = \underbrace{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}_{B_1} + i \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B_2}$

$X_1 = \left[\begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos 4t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin 4t\right] e^{3t} = \begin{pmatrix} -\sin 4t \\ -\cos 4t \end{pmatrix} e^{3t}$

$X_2 = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos 4t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin 4t\right] e^{3t} = \begin{pmatrix} \cos 4t \\ -\sin 4t \end{pmatrix} e^{3t}$

$\boxed{X = c_1 X_1 + c_2 X_2}$

3.) $\Phi^{-1}(t) = \frac{1}{3e^{-t}} \begin{pmatrix} e^{-2t} & 2e^{-2t} \\ -e^{t} & e^{t} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} e^{-t} & 2e^{-t} \\ -e^{2t} & e^{2t} \end{pmatrix}$

$\Phi^{-1}(t)F = \frac{1}{3} \begin{pmatrix} 3e^{-t} + 2e^{2t} \\ -3e^{2t} + e^{3t} \end{pmatrix}$

$\int \Phi^{-1} F = \frac{1}{3} \begin{pmatrix} -3e^{-t} + e^{2t} \\ -\frac{3}{2}e^{2t} + \frac{e^{3t}}{3} \end{pmatrix}$

$\underbrace{\Phi \int \Phi^{-1} F}_{X_p} = \frac{1}{3} \begin{pmatrix} 2e^{3t} - \frac{2}{3}e^{t} \\ -\frac{3}{2} + \frac{e^{t}}{3} - \frac{3}{2}e^{3t} + \frac{e^{4t}}{3} \end{pmatrix}$

$\boxed{X = \Phi(t) C + X_p}$

MATH 202.10 (Term 162)
Quiz 6 (Sects. 8.2 & 8.3) Duration: 30min

Name: **ID number:**

1.)(4pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} X.$

2.)(3pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 4 & 1 \\ -1 & 4 \end{pmatrix} X.$

3.)(3pts)Solve the system $X' = AX + \begin{pmatrix} e^{-t} \\ 2 \end{pmatrix}$, **given that** $\Phi(t) = \begin{pmatrix} e^{2t} & 2e^{-t} \\ e^{2t} & 3e^{-t} \end{pmatrix}$ **is a fundamental matrix of** $X' = AX$.

1.) $\begin{vmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = 0, \quad (3-\lambda)(1-\lambda)+1=0$

$\lambda^2-4\lambda+4=0, \quad \lambda=2;\ 2$

$(A-2I)K=0$

$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right), \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad x-y=0 \quad K\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

$x_2 = (tK+P)e^{2t}, \quad (A-2I)P=K$

$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & -1 & 1 \end{array}\right) \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right) \quad x-y=1 \quad P\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x_2 = \left[t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] e^{2t}$

$$\boxed{X = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2\begin{pmatrix} t+1 \\ t \end{pmatrix} e^{2t}}$$

2.) $\begin{vmatrix} 4-\lambda & 1 \\ -1 & 4-\lambda \end{vmatrix} = 0, \quad (4-\lambda)^2+1=0$

$\lambda = 4 \pm i$

$(A-(4+i)I)K=0$

$\left(\begin{array}{cc|c} 4-4-i & i & 0 \\ -1 & 4-4-i & 0 \end{array}\right) \left(\begin{array}{cc|c} -i & i & 0 \\ -1 & -i & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right) \quad x+iy=0 \quad K\begin{pmatrix} i \\ -1 \end{pmatrix}$

$K = \underbrace{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}_{B_1} + i\underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B_2}$

$X_1 = \left[\begin{pmatrix} 0 \\ -1 \end{pmatrix}\cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\sin t\right]e^{4t} = \begin{pmatrix} -\sin t \\ -\cos t \end{pmatrix} e^{4t}$

$X_2 = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\cos t + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\sin t\right]e^{4t} = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^{4t}$

$$\boxed{X = c_1 X_1 + c_2 X_2}$$

3.) $\Phi^{-1}(t) = \frac{1}{e^t}\begin{pmatrix} 3e^{-t} & -2e^{-t} \\ -e^{2t} & e^{2t} \end{pmatrix} = \begin{pmatrix} 3e^{-2t} & -2e^{-2t} \\ -e^{t} & e^{t} \end{pmatrix}$

$\Phi^{-1}F = \begin{pmatrix} 3e^{-3t} - 4e^{-2t} \\ -1 + 2e^{t} \end{pmatrix}$

$\int \Phi^{-1}F = \begin{pmatrix} -e^{-3t} + 2e^{-2t} \\ -t + 2e^{t} \end{pmatrix}$

$\underbrace{\Phi \int \Phi^{-1}F}_{X_p} = \begin{pmatrix} 6 - (1+2t)e^{-t} \\ 8 - (1+3t)e^{-t} \end{pmatrix}$

$$\boxed{X = \Phi(t)C + X_p}$$