MATH 202.5 (Term 162)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(x-3)y''+y'-2y=0$.

2.)(2pts)Find the indicial roots of the DE $2x^2y''+x(x^5+1)y'-(3+x^2)y=0$ **at** $x=0$.

3.)(3pts)Find a relation of recurrence satisfied by c_n, **where** $y=\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{3}}$ **is solution of the DE** $3xy''+2y'+y=0$.

1.) $y=\sum_{n=0}^{\infty}c_nx^n$, $|x|<\infty$

$$(x-3)\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}+\sum_{n=1}^{\infty}nc_nx^{n-1}-2\sum_{n=0}^{\infty}c_nx^n=0$$

$$-6c_2+c_1-2c_0+\sum_{k=1}^{\infty}\Big[-3(k+1)(k+2)c_{k+2}+(k+1)^2c_{k+1}-2c_k\Big]x^k=0$$

$$\begin{cases} c_2=\dfrac{c_1-2c_0}{6} \\ c_{k+2}=\dfrac{(k+1)^2c_{k+1}-2c_k}{3(k+1)(k+2)}, \quad k=1,2,3, \end{cases}$$

Case1: $\boxed{c_0=0,\ c_1\neq 0}$

$c_2=\frac{c_1}{6}$, $c_3=-\frac{2}{27}c_1$,

$$y=c_1\big(\underbrace{x+\frac{x^2}{6}-\frac{2}{27}x^3+\cdots}_{y_1}\big)$$

Case2: $\boxed{c_0\neq 0,\ c_1=0}$

$c_2=-\frac{c_0}{3}$, $c_3=-\frac{2}{27}c_0$, $c_4=$

$$y=c_0\big(\underbrace{1-\frac{x^2}{3}-\frac{2}{27}x^3+\cdots}_{y_2}\big)$$

$\boxed{y=c_1y_1+c_2y_2 \text{ is the general solution}}$

2.) $p(x)=\frac{1}{2}(1+x^5)$, $q(x)=-\frac{1}{2}(3+x^2)$

$r(r-1)+\frac{r}{2}-\frac{3}{2}=0$

$2r^2-r-3=0$, $r=-1, 3/2$.

3.) $$3x\sum_{n=0}^{\infty}(n+\tfrac{1}{3})(n-\tfrac{2}{3})c_nx^{n-\frac{5}{3}}+2\sum_{n=0}^{\infty}(n+\tfrac{1}{3})c_nx^{n-\frac{2}{3}}+\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{3}}=0$$

$$x^{\frac{1}{3}}\Big[3\sum_{n=0}^{\infty}(n+\tfrac{1}{3})(n-\tfrac{2}{3})c_nx^{n-1}+2\sum_{n=0}^{\infty}(n+\tfrac{1}{3})c_nx^{n-1}+\sum_{n=0}^{\infty}c_nx^n\Big]=0$$

$$\sum_{k=0}^{\infty}3(k+\tfrac{4}{3})(k+\tfrac{1}{3})c_{k+1}x^k+\sum_{k=0}^{\infty}2(k+\tfrac{4}{3})c_{k+1}x^k+\sum_{k=0}^{\infty}c_kx^k=0$$

$$\sum_{k=0}^{\infty}\big[(3k+4)(k+1)c_{k+1}+c_k\big]x^k=0$$

$$\boxed{c_{k+1}=-\frac{c_k}{(k+1)(3k+4)}, \quad k=0,1,2}$$

MATH 202.10 (Term 162)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: **ID number:**

1.)(5pts)Find 2 powers series solutions of the DE: $(x+2)y''-2y'-y=0$.

2.)(2pts)Find the indicial roots of the DE $3x^2y''-x(1-x^4)y'+(1-x)y=0$ **at** $x=0$.

3.)(3pts)Find a relation of recurrence satisfied by c_n**, where** $y=\sum_{n=0}^{\infty}c_nx^{n-\frac{1}{3}}$ **is solution of the DE** $3xy''+4y'-y=0$.

1.) $y=\sum_{n=0}^{\infty}c_nx^n$, $|x|<\infty$

$$(x+2)\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}-2\sum_{n=1}^{\infty}nc_nx^{n-1}-\sum_{n=0}^{\infty}c_nx^n=0$$

$$4c_2-2c_1-c_0+\sum_{k=1}^{\infty}\Big[2(k+1)(k+2)c_{k+2}+(k+1)(k-2)c_{k+1}-c_k\Big]x^k=0$$

$$\begin{cases} c_2=\dfrac{2c_1+c_0}{4} \\ c_{k+2}=\dfrac{-(k+1)(k-2)c_{k+1}+c_k}{2(k+1)(k+2)}, & k=1,2, \end{cases}$$

Case1: $\boxed{c_0=0,\ c_1\neq 0}$

$c_2=\frac{c_1}{2}$; $c_3=\frac{c_1}{6}$, $c_4=\frac{c_1}{48}$

$$y=c_1\underbrace{\left(x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{48}+\cdots\right)}_{y_1}$$

Case2: $\boxed{c_0\neq 0,\ c_1=0}$

$c_2=\frac{c_0}{4}$, $c_3=\frac{c_0}{24}$, $c_4=\frac{c_0}{96}$

$$y=c_0\underbrace{\left(1+\frac{x^2}{4}+\frac{x^3}{24}+\frac{x^4}{96}+\cdots\right)}_{y_2}$$

$\boxed{y=c_1y_1+c_2y_2 \text{ is the general solution}}$

2.) $p(x)=-\frac{1}{3}(1-x^4)$, $q(x)=\frac{1}{3}(1-x)$

$$r(r-1)-\frac{r}{3}+\frac{1}{3}=0$$

$3r^2-4r+1=0$, $r=\frac{1}{3},1$.

3.) $$3x\sum_{n=0}^{\infty}\left(n-\tfrac{1}{3}\right)\left(n-\tfrac{4}{3}\right)c_nx^{n-\frac{7}{3}}+4\sum_{n=0}^{\infty}\left(n-\tfrac{1}{3}\right)c_nx^{n-\frac{4}{3}}-\sum_{n=0}^{\infty}c_nx^{n-\frac{1}{3}}=0$$

$$x^{-\frac{1}{3}}\left[3\sum_{n=0}^{\infty}\left(n-\tfrac{1}{3}\right)\left(n-\tfrac{4}{3}\right)c_nx^{n-1}+\sum_{n=0}^{\infty}4\left(n-\tfrac{1}{3}\right)c_nx^{n-1}-\sum_{n=0}^{\infty}c_nx^n\right]=0$$

$$\sum_{k=0}^{\infty}3\left(k+\tfrac{2}{3}\right)\left(k-\tfrac{1}{3}\right)c_{k+1}x^k+\sum_{k=0}^{\infty}4\left(k+\tfrac{2}{3}\right)c_{k+1}x^k-\sum_{k=0}^{\infty}c_kx^k=0$$

$$\sum_{k=0}^{\infty}\Big[(3k+2)(k+1)c_{k+1}-c_k\Big]x^k$$

$$\boxed{c_{k+1}=\frac{c_k}{(k+1)(3k+2)},\ k=0,1,2}$$