

MATH 202.5 (Term 162)

Quiz 4 (Sects. 4.6 & 4.7)

Duration: 20min

Name: _____

ID number: _____

1.) (5pts) Solve the DE: $y'' - \frac{1}{4}y = \frac{1}{e^{-x}+1}$.

2.) (5pts) Solve the DE: $x^3y''' + 2xy' - 2y = 0$.

1) $y'' - \frac{1}{4}y = 0$

Auxiliary equation: $m^2 - \frac{1}{4} = 0$

$m = -\frac{1}{2}, m = \frac{1}{2}$

$y_c = C_1 \underbrace{e^{-\frac{x}{2}}}_{y_1} + C_2 \underbrace{e^{\frac{x}{2}}}_{y_2}, x \in (-\infty, \infty)$

$W = \begin{vmatrix} e^{-x/2} & e^{x/2} \\ -\frac{1}{2}e^{-x/2} & \frac{1}{2}e^{x/2} \end{vmatrix} = 1$

$y_p = u_1 y_1 + u_2 y_2,$

$u_1' = \frac{-y_2 f(x)}{W} = -\frac{e^{\frac{3x}{2}}}{1+e^x}$

$u_1 = -\int \frac{e^{\frac{3x}{2}}}{1+e^x} dx, u = e^{\frac{x}{2}}, du = \frac{1}{2}e^{\frac{x}{2}} dx$

$= -2 \int (1 - \frac{1}{u^2+1}) du$

$= -2(e^{x/2} - \tan^{-1} e^{x/2})$

$u_2' = \frac{y_1 f(x)}{W} = \frac{e^{-x/2}}{1+e^x}$

$u_2 = \int \frac{e^{-x/2}}{1+e^x} dx, u = e^{x/2}$

$= \int \frac{2}{1+u^2} du = 2 \tan^{-1} e^{x/2}$

$\Rightarrow y_p = -2 + 2(e^{x/2} + e^{-x/2}) \tan^{-1} e^{x/2}$
 $= -2 + 4 \cosh \frac{x}{2} \tan^{-1} e^{x/2}$

$y = y_c + y_p, x \in (-\infty, \infty)$

2.) This is a Cauchy-Euler DE

$y = x^m \Rightarrow$

$m(m-1)(m-2) + 2m - 2 = 0$

$(m-1)(m^2 - 2m + 2) = 0$

$m=1, m=1+i, m=1-i$

$\Rightarrow y = C_1 x + C_2 x \cos \ln x + C_3 x \sin \ln x, x \in (0, \infty)$

MATH 202.10 (Term 162)

Quiz 4 (Sects. 4.6 & 4.7)

Duration: 20min

Name: _____

ID number: _____

1.) (5pts) Solve the DE: $y'' + 3y' + 2y = \cos e^x$.

2.) (5pts) Solve the DE: $x^3 y''' - 2x^2 y'' + 4y = 0$.

1.) $y'' + 3y' + 2y = 0$

Auxiliary equation:

$$m^2 + 3m + 2 = 0, \quad m = -1, m = -2.$$

$$y_c = C_1 e^{-x} + C_2 e^{-2x}, \quad x \in (-\infty, \infty).$$

$$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}$$

$$y_p = u_1 y_1 + u_2 y_2,$$

$$u_1' = \frac{y_2 f(x)}{W} = \frac{x \cos e^x}{e^{-3x}}$$

$$u_1 = \int x \cos e^x dx = \sin e^x$$

$$u_2' = \frac{y_1 f(x)}{W} = -e^{2x} \cos e^x$$

$$u_2 = - \int e^{2x} \cos e^x dx$$

integration by parts

$$= -e^{2x} \sin e^x - \cos e^x$$

$$\Rightarrow y_p = -e^{2x} \cos e^x$$

$$y = C_1 e^{-x} + C_2 e^{-2x} - e^{2x} \cos e^x, \quad x \in (-\infty, \infty)$$

2.) This is a Cauchy-Euler DE.

$$y = x^m$$

$$m(m-1)(m-2) - 2m(m-1) + 4 = 0$$

$$m^3 - 5m^2 + 4m + 4 = 0$$

$$(m-2)(m^2 - 3m - 2) = 0$$

$$m = 2, \quad m_2 = \frac{3 + \sqrt{17}}{2}, \quad m_3 = \frac{3 - \sqrt{17}}{2}$$

$$\Rightarrow y = C_1 x^2 + C_2 x^{\frac{3 + \sqrt{17}}{2}} + C_3 x^{\frac{3 - \sqrt{17}}{2}}, \quad x \in (0, \infty)$$