MATH 202.5 (Term 162)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: **ID number:**

1.)(5pts)Solve the DE: $ye^{2xy} + x\cos^2 2x - 1 = (\frac{1}{y^3} - xe^{2xy} + 1)\frac{dy}{dx}$.

2.)(5pts)Solve the DE: $(1-x)^2\frac{dy}{dx} - y = y^{-2/3}$.

1.) $M(x,y)\,dx + N(x,y)\,dy = 0$

$M(x,y) = ye^{2xy} + x\cos^2 2x - 1$

$N(x,y) = -\left(\frac{1}{y^3} - xe^{2xy} + 1\right)$

$M_y = e^{2xy} + 2xy\,e^{2xy}$

$N_x = e^{2xy} + 2xye^{2xy}$

$M_y = N_x \Rightarrow$ Exact DE

$$\begin{cases} \frac{\partial f}{\partial x} = ye^{2xy} + x\cos^2 2x - 1 & (1) \\ \frac{\partial f}{\partial y} = xe^{2xy} - \frac{1}{y^3} - 1 & (2) \end{cases}$$

$(1) \Rightarrow f(x,y) = \frac{e^{2xy}}{2} + \underbrace{\int x\cos^2 2x\,dx}_{x\left(\frac{1+\cos 4x}{2}\right)} - x + g(y)$

integration by parts

$f(x,y) = \frac{e^{2xy}}{2} + \frac{1}{2}\left(\frac{x^2}{2} + \frac{x\sin 4x}{4} + \frac{\cos 4x}{16}\right) - x + g(y)$

$(2) \Rightarrow xe^{2xy} + g'(y) = xe^{2xy} - \frac{1}{y^3} - 1$

$g'(y) = -\frac{1}{y^3} - 1$

$g(y) = \frac{1}{2}\frac{1}{y^2} - y$

$$\boxed{\frac{e^{2xy}}{2} + \frac{1}{2}\left(\frac{x^2}{2} + \frac{x\sin 4x}{4} + \frac{\cos 4x}{16}\right) - x - \frac{1}{2y^2} - y = C}$$

2.) $\frac{dy}{dx} - \frac{1}{(1-x)^2}y = \frac{y^{-2/3}}{(1-x)^2}$

Bernoulli's DE

$u = y^{1+\frac{2}{3}} = y^{\frac{5}{3}} \Rightarrow y = u^{\frac{3}{5}}$

$\frac{dy}{dx} = \frac{3}{5}u^{-\frac{2}{5}}\frac{du}{dx}$

$\frac{3}{5}u^{-\frac{2}{5}}\frac{du}{dx} - \frac{1}{(1-x)^2}u^{\frac{3}{5}} = \frac{u^{-2/5}}{(1-x)^2}$

$\frac{du}{dx} - \frac{5}{3}\frac{u}{(1-x)^2} = \frac{5}{3}\frac{1}{(1-x)^2}$

$e^{-\frac{5}{3}\int\frac{dx}{(1-x)^2}} = e^{-\frac{5}{3}\frac{1}{1-x}}$

$\frac{d}{dx}\left[u\,e^{-\frac{5}{3(1-x)}}\right] = \frac{5}{3}\frac{1}{(1-x)^2}e^{-\frac{5}{3}\frac{1}{1-x}}$

$u\,e^{-\frac{5}{3(1-x)}} = \frac{5}{3}\int\frac{1}{(1-x)^2}e^{-\frac{5}{3(1-x)}}\,dx$

$= -e^{-\frac{5}{3(1-x)}} + C$

$u = -1 + C\,e^{\frac{5}{3(1-x)}}$

$$\boxed{y = \left[-1 + Ce^{\frac{5}{3(1-x)}}\right]^{\frac{3}{5}},\ x \in (1, \infty)}$$

MATH 202.10 (Term 162)

Quiz 1 (Sects. 2.4 & 2.5) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the DE: $y^2e^{-x} + x\sin^2 2x - 4x^3y = (2ye^{-x} + x^4 + \ln y)\frac{dy}{dx}$.

2.)(5pts)Solve the DE: $(x+1)^3\frac{dy}{dx} + y = y^{-1/3}$.

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1) $M(x,y)\,dx + N(x,y)\,dy = 0$

$M(x,y) = y^2 e^{-x} + x\sin^2 2x - 4x^3 y$

$N(x,y) = -(2ye^{-x} + x^4 + \ln y)$

$M_y = 2ye^{-x} - 4x^3$

$N_x = -(-2ye^{-x} + 4x^3) = 2ye^{-x} - 4x^3$

$M_y = N_x \Rightarrow$ Exact DE

$$\begin{cases} \dfrac{\partial f}{\partial x} = y^2 e^{-x} + x\sin^2 2x - 4x^3 y & (1) \\[2mm] \dfrac{\partial f}{\partial y} = -(2ye^{-x} + x^4 + \ln y) & (2) \end{cases}$$

$(1) \Rightarrow f(x,y) = -y^2 e^{-x} + \underbrace{\int x\sin^2 2x\,dx}_{x\left(\frac{1-\cos 4x}{2}\right)} - x^4 y + g(y)$

integration by parts

$f(x,y) = -y^2 e^{-x} + \frac{1}{2}\left(\frac{x^2}{2} - \frac{x\sin 4x}{4} - \frac{\cos 4x}{16}\right) + g(y) - x^4 y$

$(2) \Rightarrow -x^4 - 2ye^{-x} + g'(y) = -(2ye^{-x} + x^4 + \ln y)$

$g'(y) = \ln y$

$g(y) = y\ln y - y$

$$\boxed{-y^2 e^{-x} + \frac{1}{2}\left(\frac{x^2}{2} - \frac{x\sin 4x}{4} - \frac{\cos 4x}{16}\right) - x^4 y + y\ln y - y = C}$$

2) $\dfrac{dy}{dx} + \dfrac{y}{(x+1)^3} = \dfrac{y^{-1/3}}{(x+1)^3}$

Bernoulli's DE.

$u = y^{1+\frac{1}{3}} = y^{\frac{4}{3}} \Rightarrow y = u^{3/4}$

$\dfrac{dy}{dx} = \dfrac{3}{4}u^{-\frac{1}{4}}\dfrac{du}{dx}$

$\dfrac{3}{4}u^{-\frac{1}{4}}\dfrac{du}{dx} + \dfrac{u^{3/4}}{(x+1)^3} = \dfrac{u^{-\frac{1}{4}}}{(x+1)^3}$

$\dfrac{3}{4}\dfrac{du}{dx} + \dfrac{u}{(x+1)^3} = \dfrac{1}{(x+1)^3}$

$\dfrac{d}{dx}\left[u e^{-\frac{2}{3(x+1)^2}}\right] = \dfrac{4}{3}\dfrac{1}{(x+1)^3}e^{-\frac{2}{3}\frac{1}{(x+1)^2}}$

$u e^{-\frac{2}{3(x+1)^2}} = \dfrac{4}{3}\displaystyle\int \frac{1}{(x+1)^3}e^{-\frac{2}{3}\frac{1}{(x+1)^2}}\,dx$

$= e^{-\frac{2}{3}\frac{1}{(x+1)^2}} + C$

$u = 1 + Ce^{\frac{2}{3}\frac{1}{(x+1)^2}}$

$$\boxed{y = \left[1 + Ce^{\frac{2}{3}\frac{1}{(x+1)^2}}\right]^{\frac{3}{4}},\ x \in (-1, \infty)}$$