MATH 202.5 (Term 162)
Quiz 1 (Sects. 1.2, 2.2 & 2.3) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the IVP: $\begin{cases} y\frac{dy}{dx} = (y-3)(y+1)\cos x \sin^3 x \\ y(0) = 1. \end{cases}$

2.)(5pts)Solve the DE: $(x+1)\frac{dy}{dx} + y = \frac{x+1}{x^2+1}$.

1.)

$$\int \frac{y}{(y-3)(y+1)}\,dy = \int \cos x \sin^3 x\,dx$$

$$\wedge$$

$$= \frac{a}{y-3} + \frac{b}{y+1}$$

$$\Rightarrow y = a(y+1) + b(y-3)$$

$y=-1 \Rightarrow b = 1/4$

$y=3 \Rightarrow a = 3/4$

$$\int \left(\frac{3/4}{y-3} + \frac{1/4}{y+1}\right) dy = \frac{\sin^4 x}{4} + C$$

$$\frac{3}{4}\ln|y-3| + \frac{1}{4}\ln|y+1| = \frac{\sin^4 x}{4} + C$$

$$\ln|(y-3)^3(y+1)| = \sin^4 x + C$$

$$(y-3)^3(y+1) = C\,e^{\sin^4 x}$$

$y(0)=1 \Rightarrow -16 = C$

$$\boxed{(y-3)^3(y+1) = -16\,e^{\sin^4 x}, \quad x \in (-\infty, \infty)}$$

2.)

$$\frac{dy}{dx} + \frac{1}{x+1}y = \frac{1}{x^2+1}$$

$$e^{\int \frac{dx}{x+1}} = e^{\ln|x+1|} = x+1, \quad x > -1$$

$$\frac{d}{dx}\left[y(x+1)\right] = \frac{x+1}{x^2+1}$$

$$y(x+1) = \int \frac{x+1}{x^2+1}\,dx$$

$$= \int \left(\frac{1}{2}\frac{2x}{x^2+1} + \frac{1}{x^2+1}\right) dx$$

$$= \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C$$

$$\boxed{y = \frac{1}{x+1}\left(\frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C\right), \quad x \in (-1, \infty)}$$

MATH 202.10 (Term 162)
Quiz 1 (Sects. 1.2, 2.2 & 2.3) **Duration: 20min**

Name: **ID number:**

1.)(5pts)Solve the IVP: $\begin{cases} y\frac{dy}{dx} = (y+2)(y-1)\sec^2 x \tan^2 x \\ y(0) = 2. \end{cases}$

2.)(5pts)Solve the DE: $x\frac{dy}{dx} + y = \frac{x}{(x+1)^2}$.

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1) $\int \frac{y}{(y+2)(y-1)}\,dy = \int \sec^2 x \tan^2 x\,dx$

$\wedge$

$= \frac{a}{y+2} + \frac{b}{y-1}$

$\Rightarrow y = a(y-1) + b(y+2)$

$y=1 \Rightarrow b = 1/3$

$y=-2 \Rightarrow a = 2/3$

$\int \left(\frac{2/3}{y+2} + \frac{1/3}{y-1}\right)dy = \frac{\tan^3 x}{3} + C$

$\frac{2}{3}\ln|y+2| + \frac{1}{3}\ln|y-1| = \frac{\tan^3 x}{3} + C$

$\ln|(y+2)^2(y-1)| = \tan^3 x + C$

$(y+2)^2(y-1) = Ce^{\tan^3 x}$

$y(0)=2 \Rightarrow 8 = C$

$\boxed{(y+2)^2(y-1) = 8e^{\tan^3 x},\ x \in (-\pi/2, \pi/2)}$

2) $\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{(x+1)^2}$

$e^{\int \frac{dx}{x}} = e^{\ln|x|} = x,\ x>0$

$\frac{d}{dx}[xy] = \frac{x}{(x+1)^2}$

$xy = \int \frac{x}{(x+1)^2}\,dx$

$= \int \left(\frac{1}{x+1} + \frac{1}{(x+1)^2}\right)dx$

$= \ln|x+1| - \frac{1}{x+1} + C$

$\Rightarrow \boxed{y = \frac{1}{x}\left(\ln(x+1) - \frac{1}{x+1} + C\right),\ x>0}$