King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 202 - Final Exam - Term 162

Duration: 180 minutes

Name: KEY ID Number: ______

Section Number: ______ Serial Number: ______

Class Time: ______ Instructor's Name: ______

Instructions:

1. Calculators and Mobiles are not allowed.
2. Write legibly.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 13 pages of problems (Total of 16 Problems)
5. **DE means differential equation.**

Question # Number	Points	Maximum Points
1		10
2		10
3		8
4		5
5		11
6		10
7		13
8		8
9		8
10		4
11		13
12		8
13		8
14		5
15		8
16		11
Total		140

1. **[10 points]** Show that the DE

$$2x \sin y \, dx + (x^2 \cos y + 1) \, dy = 0$$

is exact and solve it.

$$\frac{\partial}{\partial y}(2x\sin y) = 2x\cos y = \frac{\partial}{\partial x}(x^2\cos y + 1) = 2x\cos y$$

$\Rightarrow$ The DE is exact. 2 pts

$$\frac{\partial F}{\partial x} = 2x \sin y \Rightarrow F(x,y) = x^2 \sin y + g(y)$$ 2 pts

$$\frac{\partial F}{\partial y} = x^2 \cos y + g'(y) = x^2 \cos y + 1$$ 2 pts

$$\Rightarrow \quad g'(y) = 1 \Rightarrow g(y) = y$$ 2 pts

$\Rightarrow F(x,y) = x^2 \sin y + y.$

Hence a family of solutions is

$$x^2 \sin y + y = C$$ 2pts

2. **[10 points]** Show that the DE

$$(t^2+1)\,y' = 4ty + 4t\,\sqrt{y}$$

is of Bernoulli's type, then solve it.

The given DE is of Bernoulli's type since it can be written as

$$(t^2+1)\frac{dy}{dt} - 4ty = 4t\sqrt{y}$$

or $$(t^2+1)\,y^{-1/2}\frac{dy}{dt} - 4t\,y^{1/2} = 4t.$$

Let $u = y^{1/2} \Rightarrow \dfrac{du}{dt} = \dfrac{1}{2}y^{-1/2}\dfrac{dy}{dt}$ (2 pt)

The DE transforms into

$$2(t^2+1)\frac{du}{dt} - 4tu = 4t$$

$$\Rightarrow \frac{du}{dt} - \frac{2t}{t^2+1}u = \frac{2t}{t^2+1}$$ (2 pts)

The integrating factor for this linear equation is

$$\mu(x) = e^{\int \frac{-2t}{t^2+1}dt} = \frac{1}{1+t^2}$$ (2 pts)

Integrating $\dfrac{d}{dt}\left(\dfrac{1}{1+t^2}u\right) = \dfrac{2t}{(t^2+1)^2}$

gives $\dfrac{1}{1+t^2}u = \dfrac{-1}{t^2+1} + C$ (2 pts)

So a solution of the given DE is

$$\sqrt{y} = -1 + C(t^2+1)$$

or $$y = \left[-1 + C(t^2+1)\right]^2$$ (2 pts)

3. **[8 points]** Solve the DE:

$$\left(x^2 \sin \frac{y^2}{x^2} - 2y^2 \cos \frac{y^2}{x^2}\right) dx + 2xy \cos \frac{y^2}{x^2}\, dy = 0.$$

The DE is homogeneous. Let $y = xv$, then $dy = x\,dv + v\,dx$, so after substituting, the given DE becomes (2 pts)

$$(x^2 \sin v^2 - 2x^2 v^2 \cos v^2)\,dx + 2x^2 v \cos v^2 (x\,dv + v\,dx) = 0$$

$$\Rightarrow \quad \sin v^2\, dx + 2xv \cos v^2\, dv = 0 \quad \text{(2 pts)}$$

$$\Rightarrow \quad \frac{1}{x}\,dx + 2v \cot v^2\, dv = 0 \quad \text{(1 pt)}$$

$$\Rightarrow \quad \ln|x| + \ln|\sin v^2| = \ln|c|. \quad \text{(2 pts)}$$

Hence a solution of the given DE is

$$x \sin \frac{y^2}{x^2} = C. \quad \text{(1 pt)}$$

4. **[5 points]** Solve the DE

$$y''' + y'' + y' = 0.$$

The auxiliary equation $m^3 + m^2 + m = m(m^2 + m + 1) = 0$ has roots $m_1 = 0$, $m_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, $m_3 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$.

(3 pts)

Thus the general solution of the DE is

$$y = c_1 + e^{-\frac{1}{2}x}\left(c_2 \cos \frac{\sqrt{3}}{2}x + c_3 \sin \frac{\sqrt{3}}{2}x\right). \quad \text{(2 pts)}$$

5. **[11 points]** Find the general solution of the DE:

$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

The auxiliary equation $m^2+3m+2=(m+1)(m+2)=0$ has roots $m_1=-1$ and $m_2=-2$.

Hence $y_c = c_1 e^{-x} + c_2 e^{-2x}$ (2 pts)

We find y_p by using the method of variation of Parameters:

Let $y_p = u_1 e^{-x} + u_2 e^{-2x}$, (1 pt)

$$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}, \quad W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{1+e^x} & -2e^{-2x} \end{vmatrix} = \frac{-e^{-2x}}{1+e^x},$$

$$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{1+e^x} \end{vmatrix} = \frac{e^{x}}{1+e^x}$$ (3 pts)

Hence $u_1 = \int \frac{W_1}{W}\,dx = \int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x)$ (1 pt)

and for $u_2 = \int \frac{W_2}{W} = -\int \frac{e^{2x}}{1+e^x}\,dx$, let $t=e^x$

we get $u_2 = -e^x + \ln(1+e^x)$ (2 pts)

Hence, the general solution of the DE is

$$y = y_c + y_p = c_1 e^{-x} + c_2 e^{-2x} + e^{-x}\ln(1+e^x) + e^{-2x}\left[-e^x + \ln(1+e^x)\right]$$

(2 pts)

6. **[10 points]** Find the first three nonzero terms in each of the two linearly independent power series solutions of the DE

$$y'' + 2xy' + 2y = 0,$$

about the ordinary point $x = 0$.

Substituting $y = \sum_{n=0}^{\infty} c_n x^n$ gives,

$$\sum_{n=2}^{\infty} \underbrace{n(n-1)a_n x^{n-2}}_{k=n-2} + \sum_{n=1}^{\infty} \underbrace{2n a_n x^n}_{k=n} + \sum_{n=0}^{\infty} \underbrace{2a_n x^n}_{k=n} = 0$$ 2pts

$$\Rightarrow \sum_{k=0}^{\infty} (k+2)(k+1)a_{k+2} x^k + \sum_{k=1}^{\infty} 2k a_k x^k + \sum_{k=0}^{\infty} 2a_k x^k = 0$$

$$\Rightarrow (2a_2 + 2a_0) + \sum_{k=1}^{\infty} \left[(k+2)(k+1)a_{k+2} + 2(k+1)a_k\right] x^k = 0$$ 2pts

From this identity we conclude that

$$a_2 = -a_0 \quad \text{and} \quad a_{k+2} = -\frac{2}{k+2} a_k, \; k = 1, 2, 3, \ldots$$ 2pts

$\boxed{k=1}$ $a_3 = -\frac{2}{3} a_1$ $\boxed{k=2}$ $a_4 = -\frac{1}{2} a_2 = \frac{1}{2} a_0$

$\boxed{k=3}$ $a_5 = -\frac{2}{5} a_3 = \frac{4}{15} a_1$, and so on 2pts

Therefore $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$

$\Rightarrow$ Two linearly independent power series solutions:

$$y_1 = 1 - x^2 + \frac{1}{2} x^4 - \cdots$$

and $$y_2 = x - \frac{2}{3} x^3 + \frac{4}{15} x^5 - \cdots$$ 2pts

7. **[13 points]** Find the first three nonzero terms of the series solution of the equation $4xy'' + 2y' + y = 0$ which corresponds to the larger indicial root of the DE around $x = 0$.

Clearly $x=0$ is a regular singular point.

Substituting $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ gives

$$\sum_{n=0}^{\infty} 4(n+r)(n+r-1)a_n x^{n+r-1} + \sum_{n=0}^{\infty} 2(n+r)a_n x^{n+r-1} + \sum_{n=0}^{\infty} a_n x^{n+r} = 0$$ 2pts

$$\Rightarrow x^r \left[4r(2r-1)a_0 x^{-1} + \sum_{n=1}^{\infty} \left[4(n+r)(n+r-1) + 2(n+r)\right] a_n x^{n-1} + \sum_{n=1}^{\infty} a_{n-1} x^{n-1} = 0 \right.$$ 3pts

Hence, the indicial equation is $r(2r-1) = 0$ 2pts

and the indicial roots are $r_1 = 0$ and $r_2 = \frac{1}{2}$ 1pt

We also get $a_n = \dfrac{-1}{2(n+r)(2n+2r-1)} a_{n-1}$, $n = 1, 2, 3, \ldots$ 2pts

For $\boxed{r = \frac{1}{2}}$ $a_n = \dfrac{-1}{2n(2n+1)} a_{n-1}$, $n = 1, 2, 3, \ldots$

$$\Rightarrow a_1 = -\frac{1}{6} a_0, \quad a_2 = \frac{-1}{20} a_1 = \frac{1}{120} a_0, \ldots$$ 2pts

Thus for $r = \frac{1}{2}$ we obtain the solution

$$y = x^{1/2}\left(1 - \frac{1}{6}x + \frac{1}{120}x^2 - \cdots\right)$$ 1pt

8. [**8 points**] Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$$

and corresponding eigenvectors.

From the characteristic equation

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)^2 = 0 \quad \text{2 pts}$$

we see that the eigenvalues are $\left.\begin{array}{l}\lambda_1 = 1 \\ \text{and } \lambda_2 = \lambda_3 = 2\end{array}\right\}$ 2 pts

For $\boxed{\lambda_1 = 1}$ $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \Rightarrow k_2 = k_3 = 0,$

and k_1 is free $\Rightarrow$ an eigenvector associated with $\lambda_1 = 1$ is $K_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 2 pts

For $\boxed{\lambda_2 = \lambda_3 = 2}$ $\left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right) \Longrightarrow$

$k_1 = k_2 = 0$ and k_3 is free $\Longrightarrow$ an eigenvector associated with $\lambda_2 = \lambda_3 = 2$ is $K_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ 2 pts

9. [**8 points**] Find and classify all singular points of the DE

$$x^2(x-1)\,y'' - 3xy' + 2y = 0.$$

Clearly the singular points are 0 and 1 (1 pt)

The standard form of the DE is

$$y'' - \frac{3}{x(x-1)}y' + \frac{2}{x^2(x-1)}y = 0 \Rightarrow$$

$$P(x) = -\frac{3}{x(x-1)} \text{ and } Q(x) = \frac{2}{x^2(x-1)}$$ (2 pts)

For $\boxed{x=0}$ $xP(x) = -\frac{3}{x-1}$ and $x^2Q(x) = \frac{2}{x-1}$ (2 pts)
are both analytic at $x=0$

For $\boxed{x=1}$ $(x-1)P(x) = -\frac{3}{x}$ and $(x-1)^2Q(x) = \frac{2(x-1)}{x^2}$ (2 pts)
are both analytic at $x=1$

Therefore, $x=0$ and $x=1$ are regular singular points (1 pt)

10. [**4 points**] The given vectors are solutions of a system $X' = AX$. Determine whether the vectors form a fundamental set of solutions on the interval $(0,\infty)$.

$$X_1 = \begin{pmatrix} 3 \\ -3 \end{pmatrix} e^t \text{ and } X_2 = \begin{pmatrix} 5 \\ 8 \end{pmatrix} e^t + \begin{pmatrix} 10 \\ -10 \end{pmatrix} t\,e^t.$$

$$W(X_1, X_2) = \begin{vmatrix} 3e^t & 5e^t + 10te^t \\ -3e^t & 8e^t - 10te^t \end{vmatrix} = 39e^{2t} \neq 0$$ (2 pts)
for all $t \in (0,\infty)$

Therefore X_1 and X_2 are linearly independant solutions of the given system, and hence they form a fundamental set of solutions (2 pts)

11. **[13 points]** Solve the system: $X' = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} X.$

From the characteristic equation $\begin{vmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda)(3-\lambda) = 0$

we see that the eigenvalues are $\lambda_1 = 1, \lambda_2 = 2$ and $\lambda_3 = 3$ (2 pts)

For $\boxed{\lambda_1 = 1}$ $\left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right) \Rightarrow k_2 = k_3 = 0$ and k_1 free

$\Rightarrow$ an eigenvector associated with $\lambda_1 = 1$ is $K_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\Rightarrow$ a solution of the system is $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^t$ (3 pts)

For $\boxed{\lambda_2 = 2}$ $\left(\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \Rightarrow k_2 = -k_3$ and $k_1 = k_3$

$\Rightarrow$ an eigenvector associated with $\lambda_2 = 2$ is $K_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\Rightarrow$ a solution of the system is $X_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{2t}$ (3 pts)

For $\boxed{\lambda_3 = 3}$ $\left(\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \Rightarrow k_2 = 0$ and $k_1 = k_3$

$\Rightarrow$ a solution of the system is $X_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{3t}$ (3 pts)

The solutions X_1, X_2, and X_3 are linearly independent since the eigenvalues λ_1, λ_2 and λ_3 are distinct.

Therefore the general solution of the system is

$$X = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} e^{2t} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{3t}.$$ (2 pts)

12. [**8 points**] Solve the system $X' = \begin{pmatrix} 4 & -1 \\ 1 & 6 \end{pmatrix} X$

We find the eigenvalues. From

$$\begin{vmatrix} 4-\lambda & -1 \\ 1 & 6-\lambda \end{vmatrix} = \lambda^2 - 10\lambda + 25 = (\lambda-5)^2 = 0$$

we get an eigenvalue $\lambda = 5$ of mulplicity 2

2pts

For $\boxed{\lambda=5}$ $\left(\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right) \Rightarrow k_1 = -k_2 \Rightarrow$

an eigenvector associated with $\lambda = 5$ is

$K_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and one solution is $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{5t}$

2pts

To get another linearly independent solution

we solve $(A - 5I)P = K \Rightarrow \left(\begin{array}{cc|c} -1 & -1 & 1 \\ 1 & 1 & -1 \end{array}\right)$ 1pt

$\Rightarrow P_1 + P_2 = -1$. Taking $P_1 = 0$ we find $P_2 = -1$

. So a second linearly independent solution is

$$X_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} t e^{5t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{5t}.$$

2 pts

The general solution is

$$X = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{5t} + c_2 \left[\begin{pmatrix} 1 \\ -1 \end{pmatrix} t e^{5t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{5t} \right]$$ 1pt

13. **[8 points]** Given that $\lambda_1 = 4 + i$ and $\lambda_2 = 4 - i$ are the eigenvalues of the matrix of coefficients of the system

$$\frac{dx}{dt} = 5x + y$$
$$\frac{dy}{dt} = -2x + 3y.$$

Find the general solution of the system.

For $\boxed{\lambda_1 = 4+i}$ $\left(\begin{array}{cc|c} 5-4-i & 1 & 0 \\ -2 & 3-4-i & 0 \end{array}\right)$

$\Rightarrow \left(\begin{array}{cc|c} 1-i & 1 & 0 \\ -2 & -1-i & 0 \end{array}\right) \Rightarrow k_2 = (-1+i)k_1$ 2 pts

$\Rightarrow$ an eigenvector associated with λ_1 is

$$K = \begin{pmatrix} 1 \\ -1+i \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ 2 pts

Hence, the general solution is

$$X = c_1\left[\begin{pmatrix} 1 \\ -1 \end{pmatrix}\cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\sin t\right]e^{4t}$$ 2 pts

$$+ c_2\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\cos t + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\sin t\right]e^{4t}$$ 2 pts

OR $$X = c_1\begin{pmatrix} \cos t \\ -\cos t - \sin t \end{pmatrix}e^{4t} + c_2\begin{pmatrix} \sin t \\ \cos t - \sin t \end{pmatrix}e^{4t}$$

14. **[5 points]** Find a linear differential operator with constant coefficients and of lowest order that will annihilate the function

$$f(x) = 1 + \sin^2 x + x \sin 2x.$$

$$f(x) = 1 + \tfrac{1}{2}(1 - \cos 2x) + x \sin 2x$$

$$= \tfrac{3}{2} - \underbrace{\tfrac{1}{2}\cos 2x + x\sin 2x}_{(D^2+4)^2} \quad \text{1pt}$$

Annihilator: $\tfrac{3}{2} \to D$, $\quad (D^2+4)^2$ 3pts

Hence, the required annihilator is

$$D\,(D^2+4)^2 \quad \text{1 pt}$$

15. **[8 points]** Find a particular solution of the system $X' = AX + \begin{pmatrix} t \\ 1 \end{pmatrix}$, given that $\Phi(t) = \begin{pmatrix} e^{-t} & -1 \\ 2e^{-t} & 2 \end{pmatrix}$ is a fundamental matrix for the homogeneous system $X' = AX$.

$$\Phi^{-1}(t) = \frac{e^t}{4}\begin{pmatrix} 2 & 1 \\ -2e^{-t} & e^{-t} \end{pmatrix} \quad \text{1 pts}$$

$$X_p = \Phi(t)\int \Phi^{-1}(t)\begin{pmatrix} t \\ 1 \end{pmatrix} dt \quad \text{1pt}$$

$$= \frac{1}{4}\Phi(t)\int \begin{pmatrix} (2t+1)e^t \\ -2t+1 \end{pmatrix} dt \quad \text{1pt}$$

$2t+1$ e^t
2 e^t
0 e^t

$$= \frac{1}{4}\begin{pmatrix} e^{-t} & -1 \\ 2e^{-t} & 2 \end{pmatrix}\begin{pmatrix} (2t-1)e^t \\ -t^2+t \end{pmatrix} \quad \text{3 pts}$$

$$= \frac{1}{4}\begin{pmatrix} 2t-1+t^2-t \\ 4t-2-2t^2+2t \end{pmatrix} = \frac{1}{4}\begin{pmatrix} t^2+t+1 \\ -2t^2+6t-2 \end{pmatrix}$$

2pts

16. (a) **[6 points]** If A is an $n \times n$ matrix such that $A^2 = I$, where I is the $n \times n$ identity matrix, then show that

$$e^{At} = I \cosh t + A \sinh t,$$

where $\cosh t = 1 + \frac{1}{2!}t^2 + \frac{1}{4!}t^4 + \ldots$ and $\sinh t = t + \frac{1}{3!}t^3 + \frac{1}{5!}t^5 + \ldots$

$$e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \frac{1}{4!}A^4t^4 + \cdots \quad \text{(2 pts)}$$

$$A^2 = I \Rightarrow A^3 = A,\ A^4 = I,\ A^5 = A, \ldots \quad \text{(1 pt)}$$

Therefore $e^{At} = I\left(1 + \frac{1}{2!}t^2 + \frac{1}{4!}t^4 + \cdots\right) + A\left(t + \frac{1}{3!}t^3 + \frac{1}{5!}t^5 + \cdots\right)$ (2 pt)

$$\Rightarrow e^{At} = I \cosh t + A \sinh t \quad \text{(1 pt)}$$

(b) **[5 points]** Use the result obtained in part(a) and matrix exponential method to solve the system $X' = AX$ where $A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$.

$$A^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = I \quad \text{(2 pts)}$$

Use Part(a), we get the solution of the system is

$$X = e^{At}C = (I \cosh t + B \sin t)\, C \quad \text{(2 pts)}$$

$$= \begin{pmatrix} \cosh t & 0 & 0 & 0 & \sinh t \\ 0 & \cosh t & 0 & \sinh t & 0 \\ 0 & 0 & \cosh t + \sinh t & 0 & 0 \\ 0 & \sinh t & 0 & \cosh t & 0 \\ \sinh t & 0 & 0 & 0 & \cosh t \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix}$$

(1 pt)