KFUPM--Term 162

Math 201 Quiz 4(a) Time: 20 minutes Date: 4- 5- 17

Name	ID Sol	Sr	Sec.	Marks:- /

Q 1. Find the local extreme values and saddle points of $f(x, y) = x^3 + 3xy + y^3$.

Sol: $f_x(x,y) = 3x^2 + 3y = 0 \Rightarrow x^2 + y = 0 \rightarrow ①$ and $f_y(x,y) = 3x + 3y^2 = 0$

$\Rightarrow x + y^2 = 0 \rightarrow ②$ ① and ② imply that $(0,0), (-1,-1)$ are c.p's.

Also $f_{xx}(x,y) = 6x$, $f_{yy}(x,y) = 6y$, $f_{xy}(x,y) = 3$

Check for (0,0): $D(0,0) = -9 < 0 \Rightarrow (0,0)$ is a saddle point.

Check for (-1,-1): $D(-1,-1) = (-6)(-6) - 9 = 36 - 9 = 27 > 0$.

and $f_{xx}(-1,-1) = -6 < 0 \Rightarrow f$ has local minimum

at $(-1,-1)$ and is 1.

Q2. Evaluate $\int_0^1 \int_0^1 \frac{y}{xy+1}\, dy\, dx$.

Sol: $\int_0^1 \int_0^1 \frac{y}{xy+1}\, dy\, dx = \int_0^1 \int_0^1 \frac{y}{xy+1}\, dx\, dy$

$= \int_0^1 \left[\ln|xy+1|\right]_0^1 dy = \int_0^1 \ln|y+1|\, dy$

$= \left[y \ln|y+1| - y + \ln|y+1|\right]_0^1$

$= \left[(y+1)\ln|y+1| - y\right]_0^1 = 2\ln 2 - 1$

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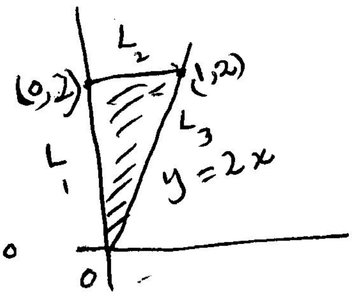
Math 201 Quiz 4(b) Time: 20 minutes Date: 4- 5- 17

Name	ID	Sr	Sec.	Marks:- /

Q 1. Find the absolute maxima and minima of $f(x,y) = 2x^2 + y^2 - 4(x+y) + 1$ on the closed triangular plate bounded by the lines: $x = 0, y = 2, y = 2x$.

Sol: On L_1 : $f(0,y) = y^2 - 4y + 1$



$\Rightarrow f'(0,y) = 2y - 4 = 0 \Rightarrow y = 2$

$\Rightarrow (0,2)$ is a c.p. on L_1

On L_2 : $f(x,2) = 2x^2 - 4x - 3 \Rightarrow f'(x,2) = 4x - 4 = 0$

$\Rightarrow x = 1 \Rightarrow (1,2)$ is a c.p. on L_2.

On L_3 : $f(x,2x) = 6x^2 - 12x + 1 \Rightarrow f'(x,2x) = 12x - 12 = 0 \Rightarrow x = 1$

$\Rightarrow (1,2)$ is a c.p.

Inside : $f_x = 4x - 4 = 0,\ f_y = 2y - 4 = 0 \Rightarrow (x,y) = (1,2)$

All c.p's are : $(0,0), (0,2), (1,2) \Rightarrow f(0,0) = 1,\ f(0,2) = -3,$ and $f(1,2) = -5$. Min. $= -5$ at $(1,2)$. Max. $= 1$ at $(0,0)$.

Q2. Evaluate $\int_0^4 \int_1^4 \left(\frac{x}{2} + \sqrt{y}\right) dy\, dx$.

Sol: $\int_0^4 \int_1^4 \left(\frac{x}{2} + \sqrt{y}\right) dy\, dx = \int_1^4 \int_0^4 \left(\frac{x}{2} + \sqrt{y}\right) dx\, dy$

$= \int_1^4 \left[\frac{x^2}{4} + x\sqrt{y}\right]_{x=0}^{4} dy = \int_1^4 (4 + 4y^{\frac{1}{2}})\, dy = \left[4y + \frac{8}{3} y^{\frac{3}{2}}\right]_1^4$

$= \left(16 + \frac{8}{3}(8)\right) - \left(4 + \frac{8}{3}\right) = \frac{92}{3}$

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Math 201 Quiz 4(c) Time: 20 minutes Date: 4- 5- 17

Name	ID Sol	Sr	Sec.	Marks: /

Q 1. Find the local extreme values and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

Sol: $f_x(x,y) = 4y - 4x^3 = 0 \Rightarrow y - x^3 = 0 \rightarrow ①$ and $f_y(x,y) = 4x - 4y^3 = 0$

$\Rightarrow x - y^3 = 0 \rightarrow ②$ ① and ② imply that $(0,0), (1,1), (-1,-1)$ are c.p's.

Also $f_{xx}(x,y) = -12x^2$, $f_{yy}(x,y) = -12y^2$ and $f_{xy}(x,y) = 4$.

Check for $(0,0)$: $D(0,0) = -16 < 0 \Rightarrow (0,0)$ is a saddle point.

Check for $(1,1)$: $D(1,1) = 144 - 16 = 128 > 0$ and $f_{xx}(1,1) = -12 < 0$

$\Rightarrow f$ has local max. at $(1,1)$ and it is: $4(1)(1) - 1^4 - 1^4 = 2$.

checking $(-1,-1)$: $D(-1,-1) = 144 - 16 = 128 > 0$ and

$f_{xx}(-1,-1) = -12 < 0 \Rightarrow f$ has local max. at $(-1,-1)$

and it is: $4(-1)(-1) - (-1)^4 - (-1)^4 = 2$.

Q2. Evaluate $\int_{-1}^{1} \int_{0}^{\pi} xy \cos y \, dy \, dx$.

Sol: $$\int_{-1}^{1}\int_{0}^{\pi} xy\cos y \, dy\, dx = \int_{-1}^{1} \Big[xy \sin y + x\cos y\Big]_{y=0}^{\pi} dx$$

$$= \int_{-1}^{1} (-2x)\, dx = -\Big[x^2\Big]_{-1}^{1} = -(1-1) = 0.$$

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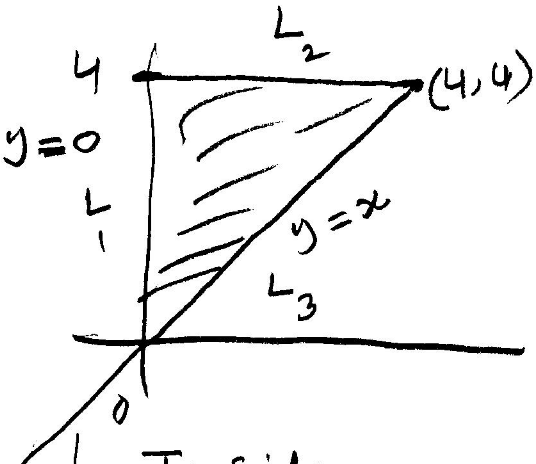
Math 201	Quiz 4(d)	Time: 20 minutes			Date: 4- 5- 17
Name	ID Solution		Sr	Sec.	Marks:- /

Q 1. Find the absolute maxima and minima of $f(x,y) = x^2 + y^2 - xy + 1$ on the closed triangular plate in the first quadrant bounded by the lines: $x = 0, y = 4, y = x$

Sol: on L_1



$f(0,y) = y^2 + 1 \Rightarrow f'(0,y) = 2y = 0$

$\Rightarrow y = 0$; $(0,0)$ is the c.p.

on L_2: $f(x,4) = x^2 - 4x + 17$

$\Rightarrow f'(x,4) = 2x - 4 = 0 \Rightarrow x = 2$

$\Rightarrow (2,4)$ is the c.p.

on L_3: $f(x,x) = x^2 + 1 \Rightarrow f'(x,x)$

$= 2x = 0 \Rightarrow x = 0 = y$.

Inside

$f_x = 2x - y = 0$

$f_y = 2y - x = 0$

$\Rightarrow x = 0 = y$

C.p's are: $(0,0)$, $(2,4)$, $(0,4)$, $(4,4)$

max. $= 17$ at $(0,4)$ and $(4,4)$

min. $= 1$ at $(0,0)$.

Q2. Evaluate $\int_1^2 \int_0^1 xye^x \, dx \, dy$.

Sol: $\int_1^2 \int_0^1 xye^x \, dx\, dy$

$= \int_0^1 \int_1^2 xye^x \, dy\, dx$

$= \int_0^1 xe^x \left[\frac{y^2}{2}\right]_1^2 dx = \frac{3}{2}\int_0^1 xe^x \, dx$

$= \frac{3}{2}\left[xe^x - e^x\right]_0^1 = \frac{3}{2}\left[(e-e) - (0-e^0)\right]$

$= \frac{3}{2}$