Solution
TAKEN 27/02/2017

King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math102/Calculus II
Quiz 1
~~three~~ Two Problems [1]

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Problem 1 (5 points)

Express the following limit as a definite integral.

$\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} \frac{2}{1+(\frac{i}{n})^2}$. (B level)

(2) $\Delta x = \frac{b-a}{n} = \frac{2}{n} \Rightarrow b-a=2$

(2) $x_i^* = a + \frac{2i}{n} \Rightarrow \frac{i}{n} = \frac{x_i^*-a}{2}$

(1) $\Rightarrow f(x_i^*) = \frac{2}{1+\left(\frac{x_i^*-a}{2}\right)^2}$

(1) $a+\frac{2}{n}$, $a+\frac{2i}{n}$, x_i^*, a, $b=a+2$

(1) Let $a=0$; $b=2$

$\Rightarrow \int_0^2 \frac{2}{1+\left(\frac{x}{2}\right)^2}\,dx$

* other solutions are possible.

Problem 2 (5 points)

Find the derivative of $g(x) = \int_{1-2x}^{1+2x} t\sin(t)\,\partial t$. (C level)

(2) $\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}\left(\int_{1-2x}^{0} t\sin(t)\,dt\right) + \frac{\partial}{\partial x}\left(\int_{0}^{1+2x} t\sin(t)\,dt\right)$

(1) Let $u = 1-2x$ & $v = 1+2x$; So $\frac{\partial u}{\partial x} = -2$ & $\frac{\partial v}{\partial x} = 2$

(2) $\frac{\partial g}{\partial x} = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$

* Be careful inversion of bounds for the first integral.

Problem 3 (5 points)

Use substitution to evaluate the indefinite integral.

(a) $\int u\sqrt{1-u^2}\,\partial u$. (B level)

(2) Let $x = 1-u^2$
$\Rightarrow \partial x = -2u\,\partial u$
$\Rightarrow -\frac{\partial x}{2} = u\,du$

(2) $\int u\sqrt{1-u^2}\,du = \int \frac{\sqrt{x}}{-2}\,\partial x$

$= -\frac{1}{2}\cdot\frac{2}{3}x^{3/2} + C = -\frac{1}{3}x^{3/2} + C$ (1)

$= -\frac{1}{3}(1-u^2)^{3/2} + C$

(b) $\int \frac{\cos(x)}{\sin^2(x)}\,\partial x$. (A level)

(2) Let $u = \sin(x)$
$\frac{\partial u}{\partial x} = \cos(x)$
$\partial u = \cos(x)\,\partial x$

(2) $\int \frac{\cos(x)}{\sin^2(x)}\,\partial x = \int \frac{\partial u}{u^2} = -\frac{1}{u} + C$ (1)

$= -\frac{1}{\sin(x)} + C$

OR $= -\csc(x) + C$

[1] The quiz lasts 30 minutes.