KEY Solution

King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math102/Calculus II
Quiz 2
Three Problems [1]

Problem 1 (5 points)

Use substitution to evaluate the indefinite integral.

$\int_e^{e^4} \frac{\partial x}{x\sqrt{\ln x}}$. (B level)

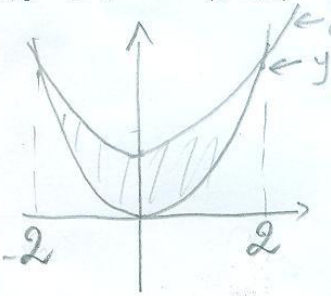
Let $u = \ln x \Rightarrow \int_1^4 \frac{\partial u}{\sqrt{u}} = 2\left[\sqrt{u}\right]_1^4 = 2(2) - 2(1) = 2$

$\partial u = \frac{1}{x}\partial x$ or $= 2\left[\sqrt{\ln x}\right]_e^{e^4} = 2.$

Problem 2 (5 points)

Skectch the region enclosed by the given curves and find its area.

$y = 2x^2$, $y = 4 + x^2$ (C level)



Intersection: $2x^2 = 4 + x^2$

$x^2 = 4 \Rightarrow x = \pm 2$

→ Even symetric functions.

$\Rightarrow A = \int_{-2}^{2} (4 + x^2) - (2x^2)\,\partial x = 2\int_0^2 4 + x^2 - 2x^2\,\partial x$

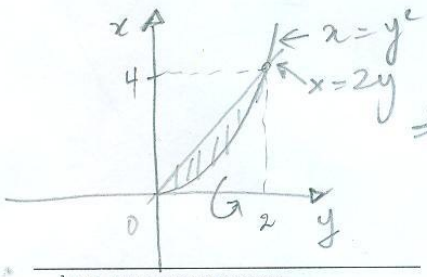
$= 2\left[4x - \frac{x^3}{3}\right]_0^2 = 2\left(8 - \frac{8}{3}\right)$

$= 32/3$

Problem 3 (5 points)

Skectch the region enclosed by the given curves and find the volume of the solid obtained by rotating the region about the y-axis.

$x = y^2$, $x = 2y$ (A level)



$\Rightarrow V = \int_0^2 \pi\left[(2y)^2 - (y^2)^2\right]\partial y$

$= \pi \int_0^2 4y^2 - y^4\,\partial y$

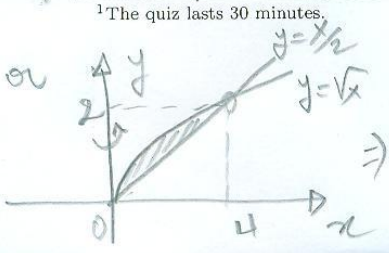
$= \pi\left(\frac{4}{3}y^3 - \frac{y^5}{5}\right)_0^2 = \pi\left[\frac{32}{3} - \frac{32}{5}\right]$

$= \pi\left[\frac{160 - 96}{15}\right]$

$= \frac{64\pi}{15}$ u.v

[1] The quiz lasts 30 minutes.

or



⇒