

Solution
TAKEN 27/10/2017

King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math102/Calculus II

Quiz 1

three Two Problems ¹

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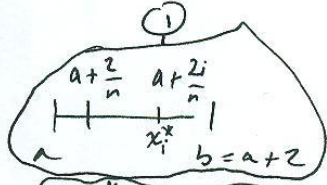
Problem 1 (5 points)

Express the following limit as a definite integral.

$$\lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \frac{2}{1 + (\frac{i}{n})^2} \quad (\text{B level})$$

① $\Delta x = \frac{b-a}{n} = \frac{2}{n} \Rightarrow b-a=2$

② $x_i^* = a + \frac{2i}{n} \Rightarrow \frac{i}{n} = \frac{x_i^* - a}{2}$



① $f(x_i^*) = \frac{2}{1 + (\frac{x_i^* - a}{2})^2}$

① Let $a=0; b=2$
 $\Rightarrow \int_0^2 \frac{2}{1 + (\frac{x}{2})^2} dx$
* other solutions are possible.

Problem 2 (5 points)

Find the derivative of $g(x) = \int_{1-2x}^{1+2x} t \sin(t) dt$. (C level)

② $\frac{dg}{dx} = \frac{d}{dx} \left(\int_{1-2x}^0 t \sin(t) dt \right) + \frac{d}{dx} \left(\int_0^{1+2x} t \sin(t) dt \right)$

① Let $u = 1-2x$ & $v = 1+2x$; So $\frac{\partial u}{\partial x} = -2$ & $\frac{\partial v}{\partial x} = 2$

② $\frac{dg}{dx} = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$

Problem 3 (5 points)

Use substitution to evaluate the indefinite integral.

(a) $\int u \sqrt{1-u^2} du$. (B level)

② Let $x = 1-u^2$
 $\Rightarrow dx = -2u du$
 $\Rightarrow -\frac{dx}{2} = u du$

② $\int u \sqrt{1-u^2} du = \int \sqrt{x} \frac{dx}{-2}$

① $= -\frac{1}{2} \cdot \frac{2}{3} x^{3/2} + C = -\frac{1}{3} x^{3/2} + C$

① $= -\frac{1}{3} (1-u^2)^{3/2} + C$

* Be careful inversion of bounds for the first integral.

(b) $\int \frac{\cos(x)}{\sin^2(x)} dx$. (A level)

② Let $u = \sin(x)$
 $\frac{du}{dx} = \cos(x)$
 $du = \cos(x) dx$

② $\int \frac{\cos(x)}{\sin^2(x)} dx = \int \frac{du}{u^2}$

① $= -\frac{1}{u} + C$
① $= -\frac{1}{\sin(x)} + C$
OR $= -\csc(x) + C$

¹The quiz lasts 30 minutes.