King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 162
Friday 26/5/2017
Net Time Allowed: 180 minutes

MASTER VERSION

Q	MM	V1	V2	V3	V4
1	a	b	d	d	b
2	a	a	a	b	e
3	a	c	b	b	d
4	a	c	e	e	b
5	a	c	a	b	c
6	a	e	b	a	e
7	a	e	a	e	d
8	a	e	b	c	b
9	a	a	c	c	a
10	a	e	d	a	d
11	a	b	c	d	e
12	a	b	c	d	c
13	a	d	c	a	c
14	a	e	d	d	e
15	a	c	b	a	e
16	a	d	b	b	e
17	a	d	c	b	d
18	a	e	c	b	a
19	a	d	b	d	c
20	a	c	b	e	a
21	a	c	c	e	a
22	a	b	c	a	e
23	a	e	b	d	e
24	a	d	e	c	a
25	a	d	c	d	e
26	a	e	e	a	e
27	a	d	c	e	c
28	a	e	d	a	a

Detailed Solutions

1. If $f(t) = \begin{cases} -1 & -3 \le t \leq 0 \\ \sqrt{t} & 0 \le t \le 1 \\ 2t-1 & 1 \le t \le 8 \end{cases}$

then $\int_{-1}^{3} f(t)\,dt =$

$$\int_{-1}^{0} -1\,dt + \int_{0}^{1} \sqrt{t}\,dt + \int_{1}^{3} 2t-1\,dt$$

$$= -t\Big]_{-1}^{0} + \frac{2}{3}t^{3/2}\Big]_{0}^{1} + t^2 - t\Big]_{1}^{3}$$

$$= -(0+1) + \left(\frac{2}{3} - 0\right) + \left[(9-3) - (1-1)\right]$$

$$= -1 + \frac{2}{3} + 6 = 5 + \frac{2}{3} = \frac{15+2}{3} = \frac{17}{3}$$

(a) $\frac{17}{3}$

(b) $\frac{13}{2}$

(c) 4

(d) $\frac{15}{4}$

(e) 6

2. $\int_{1}^{e} \frac{\sin(\pi \ln y)}{y}\,dy =$

$u = \pi \ln y \Rightarrow du = \frac{\pi}{y}\,dy$

$y = 1 \Rightarrow u = 0$

$y = e \Rightarrow u = \pi$

$$\int_{0}^{\pi} \sin u \cdot \frac{1}{\pi}\,du$$

$$= \frac{1}{\pi} \cdot -\cos u\Big]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left(1 - (-1)\right) = \frac{2}{\pi}$$

(a) $\frac{2}{\pi}$

(b) 0

(c) $\frac{-1}{\pi}$

(d) $\frac{3}{\pi}$

(e) $-\frac{4}{\pi}$

3. If $F(x) = \int_3^{2x+1} \frac{1}{2\ln t}\,dt$, then $F'(1) =$

$$F'(x) = \frac{1}{2\ln(2x+1)} \cdot \frac{d}{dx}(2x+1)$$

$$= \frac{1}{2\ln(2x+1)} \cdot 2 = \frac{1}{\ln(2x+1)}$$

$$F'(1) = \frac{1}{\ln 3}$$

(a) $\frac{1}{\ln 3}$

(b) $\frac{3}{\ln 3}$

(c) $\frac{-2}{\ln 3}$

(d) 0

(e) $\frac{1}{2\ln 3}$

4. $\int 18\,(6x+1)^8\,dx =$

$u = 6x+1 \Rightarrow du = 6\,dx$

$$= \int 18 \cdot u^8 \cdot \frac{1}{6}\,du$$

$$= 3\int u^8\,du$$

$$= 3 \cdot \frac{1}{9}u^9 + C$$

$$= \frac{1}{3}(6x+1)^9 + C$$

(a) $\frac{1}{3}(6x+1)^9 + C$

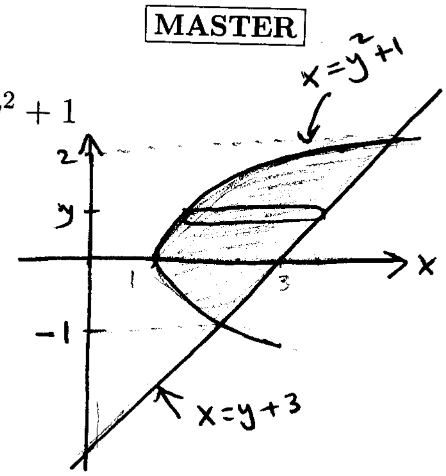
(b) $\frac{2}{3}(6x+1)^9 + C$

(c) $2\,(6x+1)^9 + C$

(d) $3\,(6x+1)^9 + C$

(e) $\frac{1}{6}(6x+1)^9 + C$

5. The **area** of the region enclosed by the curves $x = y^2 + 1$ and $x = y + 3$ is



$y^2+1 = y+3$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1, 2$

(a) $\frac{9}{2}$

(b) $\frac{5}{3}$

(c) $\frac{19}{6}$

(d) $\frac{7}{2}$

(e) $\frac{17}{3}$

$$A = \int_{-1}^{2} (y+3) - (y^2+1) \, dy$$

$$= \int_{-1}^{2} y - y^2 + 2 \, dy$$

$$= \frac{1}{2}y^2 - \frac{1}{3}y^3 + 2y \Big]_{-1}^{2}$$

$$= \left(2 - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$$

$$= 8 - 3 - \frac{1}{2}$$

$$= 5 - \frac{1}{2} = \frac{10-1}{2} = \frac{9}{2}$$

6. The improper integral $\int_0^5 \frac{1}{\sqrt[3]{5-x}} \, dx$ $= \lim_{t \to 5^-} \int_0^t (5-x)^{-1/3} \, dx$

(a) converges to $\frac{3}{2}\sqrt[3]{25}$

(b) convergest to $\sqrt[3]{25}$

(c) converges to $\frac{1}{2}\sqrt[3]{25}$

(d) converges to 0

(e) diverges

$$= \lim_{t \to 5^-} -\frac{3}{2}(5-x)^{2/3} \Big]_0^t$$

$$= \lim_{t \to 5^-} \left[-\frac{3}{2}(5-t)^{2/3} + \frac{3}{2} 5^{2/3} \right]$$

$$= 0 + \frac{3}{2} \cdot \sqrt[3]{25}$$

$$= \frac{3}{2}\sqrt[3]{25}$$

7. $\int_0^{\pi/3} \sin^3\theta \sec^2\theta \, d\theta = \int_0^{\pi/3} \frac{\sin^3\theta}{\cos^2\theta} \, d\theta = \int_0^{\pi/3} \frac{1-\cos^2\theta}{\cos^2\theta} \cdot \sin\theta \, d\theta$

$u = \cos\theta \Rightarrow du = -\sin\theta \, d\theta$
$\theta = 0 \Rightarrow u = 1$
$\theta = \frac{\pi}{3} \Rightarrow u = \frac{1}{2}$

(a) $\frac{1}{2}$

(b) $\frac{5}{2}$

(c) $\frac{\pi}{2}$

(d) $\frac{5\pi}{3}$

(e) 0

$$= \int_1^{1/2} \frac{1-u^2}{u^2} \cdot -du$$
$$= \int_{1/2}^{1} (u^{-2}-1) \, du$$
$$= -u^{-1} - u \Big]_{1/2}^{1} = -\frac{1}{u} - u \Big]_{1/2}^{1}$$
$$= (-1-1) - \left(-2-\frac{1}{2}\right)$$
$$= -2 + 2 + \frac{1}{2}$$
$$= \frac{1}{2}$$

8. If $\int y^2 e^{y/2} \, dy = (Ay^2+By+C)\, e^{y/2} + D$, then $A^2+2B+C =$

(a) 4

(b) 10

(c) 6

(d) 0

(e) 5

$$\int y^2 e^{y/2} \, dy = 2y^2 e^{y/2} - 8y e^{y/2} + 16 e^{y/2} + D$$
$$= (2y^2 - 8y + 16)\, e^{y/2} + D$$

$A = 2$, $B = -8$, $C = 16$

$$A^2 + 2B + 16$$
$$= 4 - 16 + 16$$
$$= 4$$

y^2	$e^{y/2}$
$\oplus$	
$2y$	$2e^{y/2}$
$\ominus$	
2	$4e^{y/2}$
$\oplus$	
0	$8e^{y/2}$

9. $\int \dfrac{8x^2-1}{2x^3+x^2}\,dx =$

(a) $\dfrac{1}{x} + \ln(2x^2+x)^2 + C$

(b) $\dfrac{1}{x} + 2\ln|x| + 4\ln|2x+1| + C$

(c) $-\dfrac{1}{x} + \ln|x| + 4\ln|2x+1| + C$

(d) $8\ln|2x+1| - \ln|2x^3+x^2| + C$

(e) $-\dfrac{2}{x} + 4\ln|x| + 2\ln|2x+1| + C$

$$\frac{8x^2-1}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x+1}$$

$$8x^2-1 = Ax(2x+1) + B(2x+1) + Cx^2$$

$x=0 \Rightarrow \boxed{-1 = B}$

$x=-\frac{1}{2} \Rightarrow 1 = \frac{C}{4} \Rightarrow \boxed{C=4}$

$x=1 \Rightarrow 7 = 3A - 3 + 4 \Rightarrow \boxed{A=2}$

$$I = \int \frac{2}{x} - \frac{1}{x^2} + \frac{4}{2x+1}\,dx$$

$= 2\ln|x| + \frac{1}{x} + 4\cdot\frac{1}{2}\ln|2x+1| + C$

$= 2\ln|x| + \frac{1}{x} + 2\ln|2x+1| + C$

$= \frac{1}{x} + \ln x^2 + \ln(2x+1)^2 + C$

$= \frac{1}{x} + \ln(x^2\cdot(2x+1)^2) + C$

$= \frac{1}{x} + \ln(2x^2+x)^2 + C$

10. $\int_0^1 \dfrac{1}{1+\sqrt{x}}\,dx =$

(a) $2 - 2\ln 2$

(b) $\ln 2$

(c) 3

(d) $4 - 3\ln 2$

(e) 2

$u = \sqrt{x} \Rightarrow x = u^2 \Rightarrow dx = 2u\,du$

$x=0 \Rightarrow u=0$

$x=1 \Rightarrow u=1$

$\int_0^1 \frac{1}{1+u}\cdot 2u\,du = 2\int_0^1 \frac{u}{u+1}\,du$

$= 2\int_0^1 1 - \frac{1}{u+1}\,du$

$= 2\cdot \left. u - \ln|u+1| \right]_0^1$

$= 2\cdot (1-\ln 2) - (0-0)$

$= 2\cdot(1-\ln 2)$

$= 2 - 2\ln 2$

11. $\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx =$

$u = \sin^{-1} x \qquad dv = \frac{x}{\sqrt{1-x^2}}\, dx$

$du = \frac{1}{\sqrt{1-x^2}}\, dx \qquad v = -\sqrt{1-x^2}$

(a) $\frac{1}{2} - \frac{\pi\sqrt{3}}{12}$

(b) π

(c) 0

(d) $1 - \frac{\sqrt{3}}{3}$

(e) $\frac{\pi\sqrt{3}}{2}$

$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx = -\sin^{-1} x \cdot \sqrt{1-x^2} + \int 1\, dx$

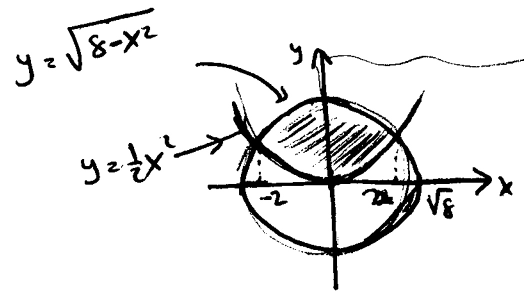
$= -\sin^{-1} x \cdot \sqrt{1-x^2} + x + C$

$\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} = -\sin^{-1} x \cdot \sqrt{1-x^2} + x \Big]_0^{1/2}$

$= -\sin^{-1}(\frac{1}{2}) \cdot \sqrt{1-\frac{1}{4}} + \frac{1}{2} - 0$

$= -\frac{\pi}{6} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{1}{2} - \frac{\pi\sqrt{3}}{12}$



pts of intersection

$x^2 + (\frac{1}{2}x^2)^2 = 8 \Rightarrow x^2 + \frac{1}{4}x^4 = 8$

$\Rightarrow 4x^2 + x^4 = 32 \Rightarrow x^4 + 4x^2 - 32 = 0$

$\Rightarrow (x^2+8)(x^2-4) = 0 \Rightarrow x = \pm 2$

12. The **area** of the region **inside** the circle $x^2 + y^2 = 8$ and **above** the parabola $y = \frac{1}{2}x^2$ is

(a) $2\pi + \frac{4}{3}$

(b) $2\pi - 3$

(c) $\pi + \frac{2}{3}$

(d) $3\pi + 1$

(e) $2\pi + \frac{1}{3}$

$A = \int_{-2}^{2} \sqrt{8-x^2} - \frac{1}{2}x^2\, dx = 2\int_0^2 \sqrt{8-x^2} - \frac{1}{2}x^2\, dx$

$= 2\int_0^2 \sqrt{8-x^2}\, dx - \int_0^2 x^2\, dx$

$= 2\int_0^2 \sqrt{8-x^2}\, dy - \frac{8}{3}$

$x = \sqrt{8}\sin\theta \Rightarrow dx = \sqrt{8}\cos\theta$

$x = 0 \Rightarrow \theta = 0;\ x = 2 \Rightarrow \theta = \frac{\pi}{4}$

$= -\frac{8}{3} + 2\int_0^{\pi/4} 8\cos^2\theta\, d\theta$

$= -\frac{8}{3} + 8\int^{\pi/4} 1 + \cos(2\theta)\, d\theta$

$= -\frac{8}{3} + 8 \cdot \theta + \frac{1}{2}\sin(2\theta)\Big]_0^{\pi/4}$

$= -\frac{8}{3} + 8 \cdot \left[(\frac{\pi}{4} + \frac{1}{2}) - 0\right]$

$= -\frac{8}{3} + 2\pi + 4$

$= 2\pi + \frac{12-8}{3} = 2\pi + \frac{4}{3}$

13. The **length** of the arc of the curve $y = \frac{2}{3}(x^2+1)^{3/2}$ from the point $\left(0, \frac{2}{3}\right)$ to the point $\left(1, \frac{4\sqrt{2}}{3}\right)$ is

$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$, $\frac{dy}{dx} = \frac{2}{3} \cdot \frac{3}{2} (x^2+1)^{1/2} \cdot 2x$

$\left(\frac{dy}{dx}\right)^2 = (x^2+1) \cdot 4x^2 = 4x^4 + 4x^2$

$1+\left(\frac{dy}{dx}\right)^2 = 4x^4+4x^2+1 = (2x^2+1)^2$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 2x^2+1$

$= \int_0^1 2x^2+1 \, dx$

$= \frac{2}{3}x^3 + x \Big]_0^1$

$= \frac{2}{3} + 1 - 0$

$= \frac{5}{3}$

(a) $\frac{5}{3}$

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{7}{8}$

(e) $\frac{8}{3}$

14. The **area of the surface** obtained by rotating the curve

$$x = \frac{1}{4}y^2 - \ln\sqrt{y}, \quad 1 \le y \le 2$$

$S = \int_1^2 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

about the x-axis is equal to

$\frac{dx}{dy} = \frac{1}{2}y - \frac{1}{2y} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^2 - \frac{1}{2} + \frac{1}{4y^2}$

$1+\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^2 + \frac{1}{2} + \frac{1}{4y^2} = \left(\frac{1}{2}y + \frac{1}{2y}\right)^2$

$S = 2\pi \int_1^2 y \cdot \left(\frac{1}{2}y + \frac{1}{2y}\right) dy$

$= 2\pi \int_1^2 \frac{1}{2}y^2 + \frac{1}{2} \, dy = \pi \int_1^2 y^2+1 \, dy$

$= \cancel{2\pi \frac{1}{6}y^3 + \frac{1}{2}}$ $= \pi \cdot \frac{1}{3}y^3 + y \Big]_1^2$

$= \pi \cdot \left[\left(\frac{8}{3}+2\right) - \left(\frac{1}{3}+1\right)\right]$

$= \pi \cdot \left(\frac{7}{3}+1\right) = \pi \cdot \frac{10}{3}$

$= \frac{10\pi}{3}$

(a) $\frac{10\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{13\pi}{3}$

(e) $\frac{2\pi}{3}$

15. The sequence $\left\{\tan^{-1}\left(\sqrt{\dfrac{60n+7}{20n+9}}\right)\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} a_n = \tan^{-1}\left(\sqrt{\tfrac{60}{20}}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

(a) converges to $\dfrac{\pi}{3}$

(b) converges to $\dfrac{\pi}{2}$

(c) converges to $\dfrac{\pi}{4}$

(d) converges to $\dfrac{5\pi}{6}$

(e) is divergent

$$S_n = \sum_{i=2}^{n}\left[\cos\left(\tfrac{2\pi}{i}\right) - \cos\left(\tfrac{2\pi}{i+1}\right)\right] = \left[\cos\left(\tfrac{2\pi}{2}\right) - \cancel{\cos\left(\tfrac{2\pi}{3}\right)}\right] + \left[\cancel{\cos\left(\tfrac{2\pi}{3}\right)} - \cancel{\cos\left(\tfrac{2\pi}{4}\right)}\right] + \left[\cancel{\cos\left(\tfrac{2\pi}{4}\right)} - \cos\left(\tfrac{2\pi}{5}\right)\right] + \cdots + \left[\cancel{\cos\left(\tfrac{2\pi}{n}\right)} - \cos\left(\tfrac{2\pi}{n+1}\right)\right]$$

16. The series $\sum_{n=2}^{\infty}\left[\cos\left(\dfrac{2\pi}{n}\right) - \cos\left(\dfrac{2\pi}{n+1}\right)\right]$ is

$$= \cos\pi - \cos\left(\tfrac{2\pi}{n+1}\right) = -1 - \cos\left(\tfrac{2\pi}{n+1}\right)$$

(a) convergent and its sum is -2

(b) convergent and its sum is -1

(c) convergent and its sum is 1

(d) convergent and its sum is 3

(e) divergent

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} -1 - \cos\left(\tfrac{2\pi}{n+1}\right) = -1 - \cos 0 = -1 - 1 = -2$$

Conv. & Sum is -2

17. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\pi}}$ is

$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n^{\pi}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ Conv. p-Series, $p = \pi > 1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\pi}}$ Conv. absolutely.

(a) absolutely convergent

(b) conditionally convergent

(c) divergent

(d) a convergent $p-$ series

(e) neither convergent nor divergent

18. The series $\sum_{n=1}^{\infty} \frac{5^{n+1}}{3^{2n}}$ is

$\sum_{n=1}^{\infty} 5\left(\frac{5}{9}\right)^n = 5\left(\frac{5}{9}\right) + 5\left(\frac{5}{9}\right)^2 + \cdots$ a Geometric Series

$a = 5\left(\frac{5}{9}\right) = \frac{25}{9}$

$r = \frac{5}{9}$, $|r| < 1 \Rightarrow$ Conv.

$\text{Sum} = \frac{a}{1-r} = \frac{\frac{25}{9}}{1-\frac{5}{9}} = \frac{25}{9-5} = \frac{25}{4}$

(a) convergent and its sum is $\frac{25}{4}$

(b) convergent and its sum is $\frac{5}{4}$

(c) convergent and its sum is $\frac{13}{2}$

(d) convergent and its sum is $\frac{17}{4}$

(e) divergent

19. The series $\sum_{n=2}^{\infty} \frac{1}{\sqrt[4]{n^5-1}}$ is

a_n $\to b_n = \frac{1}{\sqrt[4]{n^5}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt[4]{n^5}}{\sqrt[4]{n^5-1}}$

$= \lim_{n\to\infty} \sqrt[4]{\frac{n^5}{n^5-1}} = \sqrt[4]{1} = 1 > 0$

as $\sum b_n = \sum \frac{1}{n^{5/4}}$ Conv.

$(p = \frac{5}{4} > 1)$

then $\sum a_n$ Conv. by LCT

(a) convergent by the limit cmparison test.

(b) divergent by the limit cmparison test.

(c) convergent by the ratio test.

(d) divergent by the ratio test.

(e) a divergent $p-$ series.

20. If the first few terms of the **Taylor Series** of $f(x) = \sqrt[3]{x}$ about $a = 1$ are given by

$$c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + \dots$$

then $c_0 + 3c_1 + 9c_2 + 81\,c_3 =$

$f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$

(a) 6

(b) -4

(c) 0

(d) -2

(e) 8

$f(1) = 1$

$f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(1) = \frac{1}{3}$

$f''(x) = \frac{-2}{9}x^{-5/3} \Rightarrow f''(1) = -\frac{2}{9}$

$f'''(x) = \frac{10}{27}x^{-8/3} \Rightarrow f'''(1) = \frac{10}{27}$

$= 1 + \frac{1}{3}(x-1) + \frac{-2/9}{2}(x-1)^2 + \frac{\frac{10}{27}}{6}(x-1)^3 + \cdots$

$= 1 + \frac{1}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{5}{81}(x-1)^3 + \cdots$

$c_0 = 1,\ c_1 = \frac{1}{3},\ c_2 = -\frac{1}{9},\ c_3 = \frac{5}{81}$

$c_0 + 3c_1 + 9c_2 + 81c_3$

$= 1 + 1 - 1 + 5$

$= 6$

21. The series $\sum_{n=1}^{\infty} \frac{n^{2n}}{(102+3n^2)^n}$ is

$$\sqrt[n]{|a_n|} = \frac{n^2}{102+3n^2} \longrightarrow \frac{1}{3} < 1$$

Conv. by the root test

(a) convergent by the root test

(b) divergent by the root test

(c) a series for which the root test is inconclusive

(d) divergent by the test for divergence

(e) convergent by the alternating series test.

22. The series $\sum_{n=1}^{\infty} \frac{(1+\ln n)^p}{n}$ is **convergent** when $p =$

Integral Test

(a) -2

(b) -0.75

(c) -0.5

(d) -0.25

(e) 1

$$\int_1^{\infty} \frac{(1+\ln x)^p}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{(1+\ln x)^p}{x}\,dx$$

$$= \lim_{t\to\infty} \left.\frac{(1+\ln x)^{p+1}}{p+1}\right]_1^t \qquad \text{if } p \neq -1,\ p+1 \neq 0$$

$$= \lim_{t\to\infty} \frac{(1+\ln t)^{p+1}}{p+1} - \frac{1}{p+1}$$

$$= \begin{cases} -\frac{1}{p+1} & p+1<0 \implies p<-1 \\ \infty & p+1>0 \implies p>-1 \end{cases}$$

$$= \begin{cases} \text{Conv.} & p<-1 \\ \text{Div.} & p>-1 \end{cases}$$

Separately, check that the series is Div. when $p=-1$

23. The **radius of convergence** R and the **interval of convergence** I of the power series

$$\sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}}$$

are

(a) $R = \frac{1}{3}$, $I = \left[\frac{-13}{3}, \frac{-11}{3}\right)$

(b) $R = \frac{1}{3}$, $I = \left(\frac{-13}{3}, \frac{-11}{3}\right)$

(c) $R = \frac{1}{6}$, $I = \left(\frac{-1}{2}, \frac{1}{2}\right]$

(d) $R = \frac{1}{6}$, $I = \left[\frac{-1}{2}, \frac{1}{2}\right]$

(e) $R = \frac{1}{3}$, $I = \left(\frac{-13}{3}, \frac{11}{3}\right]$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{3^{n+1}(x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n(x+4)^n}\right| = 3\frac{\sqrt{n}}{\sqrt{n+1}}|x+4|$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 3|x+4|$$

• Conv. if $3|x+4|<1 \iff |x+4|<\frac{1}{3}$

$\iff -\frac{1}{3}<x+4<\frac{1}{3}$

$\iff -\frac{13}{3}<x<-\frac{11}{3}$

• end pts

$x=-\frac{13}{3}$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ Conv. Alternating Series Test

$x=\frac{-11}{3}$: $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ Div. , $p=\frac{1}{2}<1$

So $I = \left[-\frac{13}{3}, \frac{-11}{3}\right)$, $R=\frac{1}{3}$

$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

24. $$\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{6^{2n}\cdot(2n+1)!} =$$

(a) $\frac{3}{\pi}$

(b) $\frac{1}{2}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

(e) $\frac{6}{\pi}$

$$\frac{6}{\pi}\sum_{n=0}^{\infty}\frac{\pi}{6}\cdot\frac{(-1)^n\pi^{2n}}{6^{2n}\cdot(2n+1)!}$$

$$= \frac{6}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{6^{2n+1}\cdot(2n+1)!}$$

$$= \frac{6}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{\pi}{6}\right)^{2n+1}}{(2n+1)!}$$

$$= \frac{6}{\pi}\sin\left(\frac{\pi}{6}\right)$$

$$= \frac{6}{\pi}\cdot\frac{1}{2} = \frac{3}{\pi}$$

25. $\int_0^{\sqrt{2}} \frac{x^2}{e^{x^4}}\,dx =$

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{\sqrt{2}}{4n+3} \cdot \frac{2^{2n+1}}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{\sqrt{2}}{4n+3} \cdot \frac{2^{2n+1}}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{(\sqrt{2})^{4n+1}}{(4n+1)\cdot n!}$

(d) $\sum_{n=0}^{\infty}(-1)^n \frac{2}{4n+1} \cdot \frac{(\sqrt{2})^n}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{1}{(4n+3)\cdot n!}$

$x^2 e^{-x^4} = x^2 \sum_{n=0}^{\infty} \frac{(-x^4)^n}{n!} = x^2 \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{n!}$

$\int_0^{\sqrt{2}} x^2 e^{-x^4} dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{\sqrt{2}} x^{4n+2}\, dx$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \frac{x^{4n+3}}{4n+3} \Big]_0^{\sqrt{2}}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \frac{(\sqrt{2})^{4n+3}}{4n+3}$

$(\sqrt{2})^{4n+3} = \sqrt{2}\,.\,(\sqrt{2})^{4n+2} = \sqrt{2}\,.\,2^{2n+1}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \frac{\sqrt{2}\cdot 2^{2n+1}}{4n+3}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{2}}{4n+3} \cdot \frac{2^{2n+1}}{n!}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x|<1$

26. For some suitable values of x, a power series representation of $f(x) = \frac{2}{2+x^4}$ is

$f(x) = \frac{1}{1+\left(\frac{x^4}{2}\right)} = \sum_{n=0}^{\infty} \left(-\frac{x^4}{2}\right)^n \quad , \left|-\frac{x^4}{2}\right| < 1$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{2^n} \quad , \quad |x| < \sqrt[4]{2}$

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{2^n}$

(b) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{2^2}$

(c) $\sum_{n=0}^{\infty} \frac{x^{4n}}{2^n}$

(d) $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n}$

(e) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{4^n}$

27. If A is the coefficient of x^4 and B is the coefficient of x^6 in the Maclaurin series of $\sqrt{1+2x^3}$, then $A - 2B =$

$$\left(\textbf{Hint}: (1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \ldots, |x| < 1\right)$$

$$(1+2x^3)^{1/2} = 1 + \tfrac{1}{2}(2x^3) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(2x^3)^2 + \frac{\frac{1}{2}(\frac{1}{2}-\frac{1}{2})(\frac{1}{2}-2)}{3!}(2x^3)^3 + \cdots$$

$$= 1 + x^3 + \frac{-\frac{1}{4}}{2} \cdot 4x^6 + (\ )x^9 + \cdots$$

$$= 1 + x^3 - \tfrac{1}{2}x^6 + \cdots$$

$A = 0, \quad B = -\frac{1}{2}$

$A - 2B = 0 - 2(-\frac{1}{2}) = 1$

(a) 1

(b) -2

(c) 0

(d) $\frac{1}{2}$

(e) -4

28. Which one of the following statements is **TRUE**:
(C: Convergent, D: Divergent)

(a) If $\sum_{n=1}^{\infty} a_n$ is D, then $\sum_{n=1}^{\infty} |a_n|$ is D T : $\sum |a_n|$ C $\Rightarrow \sum a_n$ C

(b) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is C F : $a_n = \frac{1}{n}$

(c) If $a_n > 0$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then $\sum_{n=1}^{\infty} a_n$ is D F : C

(d) If $\sum_{n=1}^{\infty} a_n$ is D, then $\sum_{n=1}^{\infty} (-1)^n a_n$ is C F : $a_n = 2^n$

(e) If $\{|a_n|\}_{n=1}^{\infty}$ is C, then $\sum_{n=1}^{\infty} a_n$ is C. F : $a_n = \frac{1}{n}$