ANSWER KEY ✓

Q	MM	V1	V2	V3	V4
1	a	d	d	e	b
2	a	e	d	e	a
3	a	d	b	e	d
4	a	e	a	c	a
5	a	a	d	e	d
6	a	e	b	b	d
7	a	d	a	b	e
8	a	e	e	b	b
9	a	c	c	a	c
10	a	c	c	e	a
11	a	a	a	b	b
12	a	d	b	e	d
13	a	d	a	d	d
14	a	e	e	d	c
15	a	c	d	b	e
16	a	a	e	c	e
17	a	c	b	d	c
18	a	b	d	b	e
19	a	c	e	d	c
20	a	e	d	a	d

Detailed Solutions $\longrightarrow$

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 162
Wednesday 15/3/2017
Net Time Allowed: 120 minutes

MASTER VERSION

1. Using **three** rectangles and **midpoints**, the area under the graph of $f(x) = 3x - x^2$ from $x = 0$ to $x = 3$ is approximately equal to

$\Delta x = \frac{3-0}{3} = 1$

(a) $\frac{19}{4}$

(b) $\frac{17}{4}$

(c) $\frac{17}{2}$

(d) 9

(e) $\frac{8}{3}$

0 $\frac{1}{2}$ 1 $\frac{3}{2}$ 2 $\frac{5}{2}$ 3

$$A \approx f(\tfrac{1}{2})\Delta x + f(\tfrac{3}{2})\Delta x + f(\tfrac{5}{2})\Delta x$$
$$= (\tfrac{3}{2} - \tfrac{1}{4}) + (\tfrac{9}{2} - \tfrac{9}{4}) + (\tfrac{15}{2} - \tfrac{25}{4})$$
$$= \frac{5}{4} + \frac{9}{4} + \frac{5}{4}$$
$$= \frac{19}{4}$$

2. $\displaystyle\int \left(\frac{1-x}{x}\right)^2 dx =$ $\displaystyle\int \frac{1-2x+x^2}{x^2}\,dx = \int \frac{1}{x^2} - \frac{2}{x} + 1 \quad dx$

$$= -\frac{1}{x} - 2\ln|x| + x + C$$

(a) $-\frac{1}{x} - 2\ln|x| + x + C$

(b) $-\frac{1}{3}\left(\frac{1-x}{x}\right)^3 + C$

(c) $-\frac{1}{x} + x + C$

(d) $\frac{2}{x} + \ln|x| - x + C$

(e) $\frac{1}{x^2} + \frac{2}{x} + C$

3. $\int \frac{6}{x(\ln x)^4}\,dx =$

$u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$= \int \frac{6}{u^4}\,du = \int 6u^{-4}\,du$

$= 6 \cdot \frac{u^{-3}}{-3} + C$

$= -2 \cdot \frac{1}{u^3} + C$

$= \frac{-2}{(\ln x)^3} + C$

(a) $\frac{-2}{(\ln x)^3} + C$

(b) $\frac{-3}{(\ln x)^3} + C$

(c) $\frac{x}{(\ln x)^2} + C$

(d) $\frac{1}{3(\ln x)^3} + C$

(e) $\frac{6}{(\ln x)^2} + C$

4. $\int_0^1 x(\sqrt[3]{x} + 3x^2\sqrt{x})\,dx = \int_0^1 x\,x^{1/3} + 3x^3 \cdot x^{1/2}\,dx$

$= \int_0^1 x^{4/3} + 3x^{7/2}\,dx$

$= \int_0^1 x^{4/3} + 3x^{7/2}\,dx$

$= \frac{3}{7}x^{7/3} + \frac{6}{9}x^{9/2}\Big]_0^1$

$= \left(\frac{3}{7} + \frac{2}{3}\right) - 0$

$= \frac{9+14}{21} = \frac{23}{21}$

(a) $\frac{23}{21}$

(b) $\frac{21}{5}$

(c) $\frac{1}{2}$

(d) $\frac{3}{7}$

(e) $\frac{8}{5}$

5. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cos\left(1+\frac{i}{n}\right)^2 =$

$x_i^* = 1 + \frac{i}{n}$, $\Delta x = \frac{1}{n}$

$= a + i\Delta x$

$\Rightarrow a = 1$, $\Delta x = \frac{b-a}{n} = \frac{1}{n}$

$\Rightarrow a = 1$, $b = 2$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \cos(x_i^{*2})\, \Delta x$

$= \int_1^2 \cos(x^2)\, dx$

(a) $\int_1^2 \cos(x^2)\, dx$

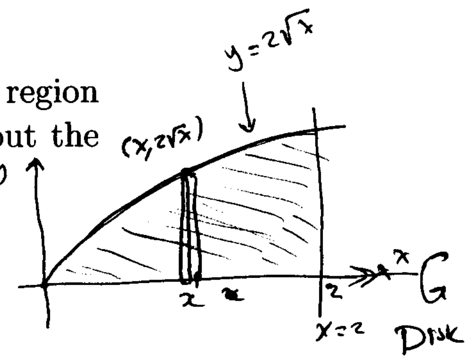
(b) $\int_1^2 \cos(1+x^2)\, dx$

(c) $\int_1^2 \cos^2 x\, dx$

(d) $\int_0^1 \cos(x^2)\, dx$

(e) $\int_0^1 \cos(1+x^2)\, dx$

6. The **volume** of the solid obtained by rotating the region bounded by the curves $y = 2\sqrt{x}$, $y = 0$, $x = 2$ about the x-axis is



(a) 8π

(b) $\frac{5\pi}{2}$

(c) 3π

(d) 6π

(e) $\frac{\pi}{2}$

$V = \pi \int_0^2 (2\sqrt{x})^2\, dx$

$= 4\pi \int_0^2 x\, dx$

$= 4\pi \cdot \left.\frac{x^2}{2}\right]_0^2$

$= 4\pi \cdot (2-0)$

$= 8\pi$

7. If $F(x) = \int_x^{x^2} e^{t^2}\, dt$ then $F'(x) =$ $e^{(x^2)^2} \cdot \frac{d}{dx}(x^2) - e^{x^2} \cdot \frac{d}{dx}(x)$

$= 2x\, e^{x^4} - e^{x^2}$

(a) $2x\, e^{x^4} - e^{x^2}$

(b) $e^{x^4} - e^{x^2}$

(c) $e^{x^4 - x^2}$

(d) $2\, e^{x^2} - e^{x}$

(e) $2\, e^{x^4} - x\, e^{x^2}$

8. $\int \frac{\tan\theta}{\sec\theta(\sec\theta - \cos\theta)}\, d\theta =$ $\int \frac{\tan\theta}{\sec^2\theta - 1}\, d\theta = \int \frac{\tan\theta}{\tan^2\theta}\, d\theta$

$= \int \frac{1}{\tan\theta}\, d\theta$

$= \int \cot\theta\, d\theta$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \ln|\sin\theta| + C$

(a) $\ln|\sin\theta| + C$

(b) $\ln|\sec\theta - \cos\theta| + C$

(c) $\sin\theta + \tan\theta + C$

(d) $-\tan\theta + \ln|\sin\theta| + C$

(e) $\cot\theta + \cos\theta + C$

9. An equation for the **tangent line** to the curve $y = \int_x^{\sqrt{3}} \sqrt{1+t^2}\, dt$ at the point with x-coordinate $\sqrt{3}$ is given by

(a) $y = -2x + 2\sqrt{3}$

(b) $y = 2x - 2\sqrt{3}$

(c) $y = \sqrt{3}x - 3$

(d) $y = 3x - 3\sqrt{3}$

(e) $y = -\sqrt{3}x + 2\sqrt{3}$

• point: $x = \sqrt{3} \Rightarrow y = \int_{\sqrt{3}}^{\sqrt{3}} \sqrt{1+t^2}\, dt = 0$

$(\sqrt{3}, 0)$

• slope: $y = -\int_{\sqrt{3}}^{x} \sqrt{1+t^2}\, dt$

$\frac{dy}{dx} = -\sqrt{1+x^2}$

slope $= \left.\frac{dy}{dx}\right|_{x=\sqrt{3}} = -\sqrt{1+3} = -2$

• Equation: $y - 0 = -2(x - \sqrt{3})$

$\Rightarrow y = -2x + 2\sqrt{3}$

10. If $f(x) = \begin{cases} 2 + \sqrt{4 - x^2} & \text{if} \quad x < 2 \\ |x - 4| & \text{if} \quad x \geq 2, \end{cases}$

then $\int_{-2}^{4} f(x)\, dx =$

(a) $2\pi + 10$

(b) $\pi - 6$

(c) $2\pi + 2$

(d) $\pi - 2$

(e) $6 + \frac{\pi}{2}$

$= \int_{-2}^{2} (2 + \sqrt{4 - x^2})\, dx + \int_{2}^{4} |x - 4|\, dx$

$= 2x\Big]_{-2}^{2} + \frac{1}{2}\pi (2)^2 + \int_{2}^{4} 4 - x \quad dx$

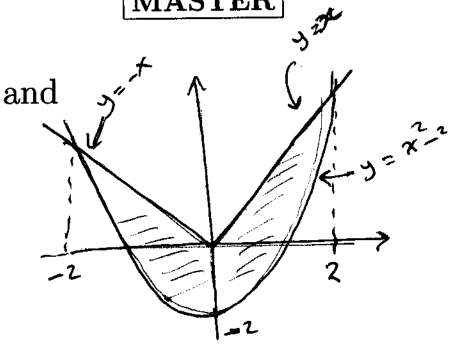
$= 8 + 2\pi + (4x - \frac{1}{2}x^2)\Big]_{2}^{4}$

$= 8 + 2\pi + (16 - 8) - (8 - 2)$

$= 8 + 2\pi + 8 - 6$

$= 2\pi + 10$

11. The **area** of the region enclosed by the curves $y = |x|$ and $y = x^2 - 2$ is



By Symmetry about the y-axis:

$$A = 2\cdot\int_0^2 [x-(x^2-2)]\,dx$$
$$= 2\cdot\int_0^2 x - x^2 + 2\,dx$$
$$= 2\cdot \left.\tfrac12 x^2 - \tfrac13 x^3 + 2x\right]_0^2$$
$$= 2\cdot\left[\left(2-\tfrac83+4\right)-0\right]$$
$$= 2\cdot\left(6-\tfrac83\right)$$
$$= 2\cdot\frac{18-8}{3} = 2\cdot\frac{10}{3} = \frac{20}{3}$$

(a) $\frac{20}{3}$

(b) $\frac{15}{16}$

(c) $\frac{25}{17}$

(d) $\frac{11}{5}$

(e) $\frac{17}{12}$

$$\Delta x = \frac{5-2}{n} = \frac{3}{n}$$
$$x_i^* = a + i\,\Delta x = 2+\frac{3i}{n}$$

12. Using n subintervals with **right endpoints**, we get

$$\int_2^5 (x^2-4)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(2+\frac{3i}{n}\right)^2-4\right]\frac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(4+\frac{12i}{n}+\frac{9i^2}{n^2}-4\right)\frac{3}{n}$$
$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{36i}{n^2}+\frac{27i^2}{n^3}\right)$$
$$= \lim_{n\to\infty}\frac{36}{n^2}\cdot\frac{n(n+1)}{2}+\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$
$$= \lim_{n\to\infty} 18\,\frac{(n+1)}{n}+\frac{9}{2}\,\frac{(n+1)(2n+1)}{n^2}$$

(a) $\lim_{n\to\infty}\left[\frac{18(n+1)}{n}+\frac{9(n+1)(2n+1)}{2n^2}\right]$

(b) $\lim_{n\to\infty}\left[\frac{9(n+1)(2n+1)}{n^2}-4n\right]$

(c) $\lim_{n\to\infty}\left[\frac{9(n+1)}{2n}-\frac{9(n+1)(2n+1)}{2n^2}\right]$

(d) $\lim_{n\to\infty}\left[\frac{12(n+1)}{n}+\frac{15(n+1)(2n+1)}{2n^2}\right]$

(e) $\lim_{n\to\infty}\left[\frac{18(n+1)}{n}+\frac{9(n+1)(2n+1)}{n^2}\right]$

$y = x^2 \Rightarrow x = \sqrt{y}$

13. The **volume** of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $x = -1$ is given by

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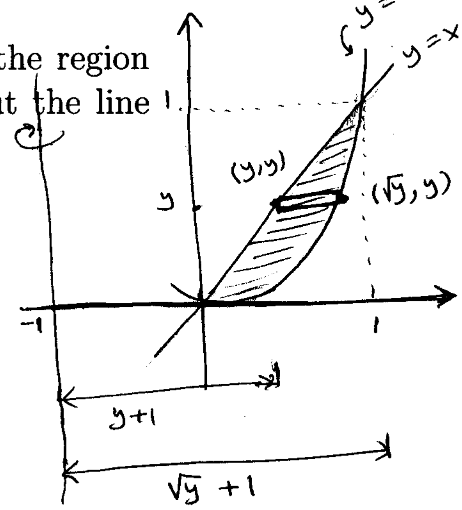


(a) $\pi \int_0^1 [(1+\sqrt{y})^2 - (1+y)^2]\, dy$

(b) $\pi \int_0^1 [y - (1+y)^2]\, dy$

(c) $\pi \int_0^1 y - y^2\, dy$

(d) $\pi \int_0^1 [(x^2+1)^2 - (x+1)^2]\, dx$.

$$V = \pi \int_0^1 [(\sqrt{y}+1)^2 - (y+1)^2]\, dy$$

(e) $\pi \int_0^1 (\sqrt{y} - y)\, dy$

14. $\displaystyle\int_{-1}^{1} \frac{\sin^3 t}{2 + \sin^2 t}\, dt =$ 0 since $f(t) = \dfrac{\sin^3 t}{2 + \sin^2 t}$ is an odd function (check?)

(a) 0

(b) $\ln 2$

(c) $2 \ln(2 + \sin 1)$

(d) $-\ln(\sin 1)$

(e) $\ln\left(\dfrac{2 + \sin 1}{2 - \sin 1}\right)$

15. If $\int_{-5}^{7} f(x)dx = -17$, $\int_{-5}^{11} f(x)\,dx = 32$, and $\int_{8}^{7} f(x)\,dx = 5$, then $\int_{11}^{8} f(x)\,dx =$

$-5 \quad 7 \quad 8 \quad 11$

$\int_{-5}^{11} f(x)dx = \int_{-5}^{7} f(x)\,dx + \int_{7}^{8} f(x)dx + \int_{8}^{11} f(x)\,dx$

$32 = -17 - 5 + \int_{8}^{11} f(x)dx$

$\int_{8}^{11} f(x)dx = 32 + 17 + 5 = 32 + 22 = 54$

$\int_{11}^{8} f(x)\,dx = -54$

(a) -54

(b) 19

(c) -60

(d) 44

(e) -50

16. The velocity $(in\, m/s)$ of a particle moving along a line is given by

$$v(t) = t^2 - 2$$

$t^2 - 2$: $+ \quad - \quad +$; $-\sqrt{2} \quad \sqrt{2}$; $0 \quad 2$

The **distance** traveled by the particle during the time interval $0 \le t \le 2$ is

$\text{Distance} = \int_0^2 |v(t)|dt = \int_0^2 |t^2-2|\,dt$

$= \int_0^{\sqrt{2}} |t^2-2|\,dt + \int_{\sqrt{2}}^{2} |t^2-2|\,dt$

$= \int_0^{\sqrt{2}} 2-t^2\,dt + \int_{\sqrt{2}}^{2} t^2-2\,dt$

$= 2t - \frac{t^3}{3}\Big]_0^{\sqrt{2}} + \frac{t^3}{3} - 2t\Big]_{\sqrt{2}}^{2}$

$= 2\sqrt{2} - \frac{2\sqrt{2}}{3} - 0 + \left(\frac{8}{3} - 4\right) - \left(\frac{2\sqrt{2}}{3} - 2\sqrt{2}\right)$

$= \frac{4\sqrt{2}}{3} + \frac{8-12}{3} - \left(\frac{-4\sqrt{2}}{3}\right)$

$= \frac{8\sqrt{2}}{3} + \frac{-4}{3} = \frac{8\sqrt{2}-4}{3}$.

(a) $\dfrac{8\sqrt{2}-4}{3}m$

(b) $\dfrac{4\sqrt{2}+2}{3}m$

(c) $\dfrac{4+\sqrt{2}}{3}m$

(d) $\dfrac{8-2\sqrt{2}}{3}m$

(e) $\dfrac{5-3\sqrt{2}}{3}m$

17. If f is a continuous function and

$$2 \le f(x) \le 5 \quad \text{for} \quad 3 \le x \le 9,$$

$\Longrightarrow f(x) > 0$ for $3 \le x \le 9$
$\Longrightarrow |f(x)| = f(x)$ for $3 \le x \le 9$

then which one of the following statements is in general **FALSE**:

(a) $\int_3^9 (\overbrace{1 - 2|f(x)|}^{1-2f(x)})\,dx \ge -10$

$-10 \le -2f(x) \le -4$
$-9 \le 1-2f(x) \le -3$
$-54 \le \int_3^9 (1-2f(x))\,dx \le -18$

(b) $\int_3^9 (3 - f(x))\,dx \ge -12$

(c) $\int_3^9 |f(x)|\,dx \ge 12$

(d) $\int_3^9 -2f(x)\,dx \le -24$

(e) $\int_3^9 (f(x))^2\,dx \ge 24$

18. $\int x^3\sqrt{x^2+1}\,dx = \int x^2\sqrt{x^2+1}\cdot x\,dx$; $u = x^2+1 \Rightarrow du = 2x\,dx$

$= \int (u-1)\sqrt{u}\cdot\frac{1}{2}du = \frac{1}{2}\int u^{3/2} - u^{1/2}\,du$

$= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C$

$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$

$= \frac{1}{5}(x^2+1)^{3/2}\left[(x^2+1) - \frac{5}{3}\right] + C$

$= \frac{1}{5}(x^2+1)^{3/2}\left(x^2 - \frac{2}{3}\right) + C$

(a) $\frac{1}{5}(x^2+1)^{3/2}\left(x^2 - \frac{2}{3}\right) + C$

(b) $\frac{1}{3}(x^2+1)^{3/2}\left(x^2 - \frac{3}{5}\right) + C$

(c) $\frac{1}{5}(x^2+1)^5 + \frac{1}{3}\left(x^2+1\right)^3 + C$

(d) $\frac{1}{5}(x^2+1)^{3/2}\left(x^2 - \frac{4}{3}\right) + C$

(e) $\frac{1}{3}(x^2+1)^{3/2}\left(\frac{3}{5}x^2 - 3\right) + C$

19. If $f(x) = x^{-1}\left[\cos\left(\frac{\pi}{4}\ln x\right)\right]^{-2}\left[4 + 5\tan\left(\frac{\pi}{4}\ln x\right)\right]^{-1/2}$,
then $\int_1^e f(x)\,dx =$

(a) $\frac{8}{5\pi}$

(b) 4

(c) $\frac{6}{5\pi}$

(d) 4π

(e) $\frac{30}{\pi}$

- $u = 4 + 5\tan\left(\frac{\pi}{4}\ln x\right) \Rightarrow du = 5\sec^2\left(\frac{\pi}{4}\ln x\right)\cdot\frac{\pi}{4}\frac{1}{x}\,dx$

$\Rightarrow \frac{4}{5\pi}\,du = \frac{1}{x\cos^2\left(\frac{\pi}{4}\ln x\right)}\,dx$

- $x = 1 \Rightarrow u = 4$ & $x = e \Rightarrow u = 4 + 5 = 9$

- $$\int_1^e f(x)dx = \int_4^9 u^{-1/2}\cdot\frac{4}{5\pi}\,du$$
$$= \frac{4}{5\pi}\cdot 2u^{1/2}\Big]_4^9$$
$$= \frac{8}{5\pi}(3-2)$$
$$= \frac{8}{5\pi}$$

20. The base of a solid is the triangular region with vertices $(0,0)$, $(1,0)$, and $(0,1)$. If the cross sections of the solid perpendicular to the x-axis are **semi-circles**, then the **volume** of the solid is

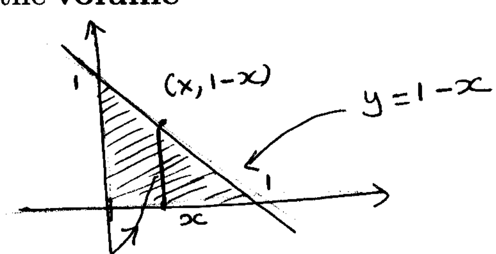
(a) $\frac{\pi}{24}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{4}$

(e) $\frac{\pi}{3}$



Diameter $= (1-x)$

radius $= \frac{1-x}{2}$

$A(x) = \frac{1}{2}\pi\left(\frac{1-x}{2}\right)^2 = \frac{\pi}{8}(1-x)^2$

$$V = \int_0^1 A(x)\,dx$$
$$= \frac{\pi}{8}\int_0^1 (1-x)^2\,dx = -\frac{\pi}{8}\cdot\frac{(1-x)^3}{3}\Big]_0^1$$
$$= -\frac{\pi}{8}\left(0 - \frac{1}{3}\right)$$
$$= \frac{\pi}{24}$$