King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Exam II
Term 162
Wednesday 19/4/2017
Net Time Allowed: 120 minutes

MASTER VERSION

1. The position of a particle along a straight line is given by $S(t) = t\,e^{-t/2}$ in the time interval $[0, 5]$ seconds. The particle is slowing down when

   (a) $0 < t < 2$ and $4 < t < 5$

   (b) $0 < t < 3$ and $4 < t < 5$

   (c) $0 < t < 2$ and $3 < t < 5$

   (d) $2 < t < 4$

   (e) $2 < t < 5$

2. If $g(x) = \big(f(x^3)\big)^2$ and $f(8) = \dfrac{1}{4}$, $f'(8) = 2$, then $g'(2) =$

   (a) 12

   (b) 24

   (c) 1

   (d) 48

   (e) 36

3. $\lim_{\theta \to 0} \left(\dfrac{2 \sin \theta}{\theta + \tan \theta} \right) =$

(a) 1

(b) 2

(c) 4

(d) ∞

(e) 0

4. An equation of the tangent line to the curve $y = x^2 \ln x$ at $(1, 0)$ is

(a) $y = x - 1$

(b) $y = x + 1$

(c) $y = x + 2$

(d) $y = x - 2$

(e) $y = \dfrac{x}{2} - \dfrac{1}{2}$

5. If $y = (2x+1)^{\sec x}$, then $y'(0) =$

(a) 2

(b) 1

(c) -1

(d) 0

(e) $\ln 2$

6. If f and g are the functions whose graphs are shown below, and let $h(x) = f(g(x))$. Then $h'(1) + h'(2) =$

(a) 4

(b) 6

(c) -2

(d) 0

(e) -6

7. A particle is moving along the hyperbola $xy = 16$. As it reaches the point $(8, 2)$, the y-coordinate is decreasing at a rate of $3\,cm/s$. How fast is the x-coordinate of the point changing at that instant?

(a) $12\,cm/s$

(b) $-12\,cm/s$

(c) $-6\,cm/s$

(d) $6\,cm/s$

(e) $0\,cm/s$

8. The curve $y = e^x + x^3 + 5x^7$ has

(a) no horizontal tangent line

(b) two horizontal tangent lines

(c) one horizontal tangent line

(d) one vertical tangent line

(e) two vertical tangent lines

9. If $x^4 + y^4 = 16$, then $y'' =$

(a) $\dfrac{-48x^2}{y^7}$

(b) $\dfrac{-3x(x^3 + y^3)}{y^7}$

(c) $\dfrac{16}{y^6}$

(d) $\dfrac{3x^2\,y^3 - 3x^3\,y^2}{y^6}$

(e) $\dfrac{3x^2y^3 + 3x^3y^2}{y^6}$

10. The position of a particle is given by the equation $s(t) = \cos\left(\dfrac{\pi}{4}\,t\right)$, $0 \le t \le 10$, where t is measured in seconds and s in feet. The total distance traveled by the particle during the first 8 seconds is

(a) $4\,ft$

(b) $8\,ft$

(c) $6\,ft$

(d) $2\,ft$

(e) $10\,ft$

11. $\dfrac{d}{dx}\left[\tan^{-1}(\cot x)\right] =$

(a) -1

(b) $\dfrac{-1}{\csc^2 x}$

(c) 0

(d) $-\tan^{-2}(\cot x)\,(\csc x \cot x)$

(e) $\csc^2 x$

12. $\lim\limits_{x\to\infty}\left(1+\dfrac{5}{3x}\right)^{x} =$

(a) $e^{5/3}$

(b) $e^{3/5}$

(c) 1

(d) 5

(e) 0

13. The equation of the normal line to the graph of the curve $y = \dfrac{1+\sin x}{x+\cos x}$ at $\left(\pi, \dfrac{1}{\pi-1}\right)$ is

(a) $y = \dfrac{(\pi-1)^2}{\pi}(x-\pi) + \dfrac{1}{\pi-1}$

(b) $y = \dfrac{\pi}{(\pi-1)^2}(x-\pi) + \dfrac{1}{\pi-1}$

(c) $y = (\pi-1)^2(x-\pi) + \pi$

(d) $y = -(\pi-1)^2(x-\pi) + \dfrac{1}{\pi-1}$

(e) $y = -\dfrac{1}{(\pi-1)^2}(x-\pi) + \dfrac{1}{\pi-1}$

14. If $f(x) = \cos(x)$, then $f^{(42)}(0) =$

(a) -1

(b) 0

(c) -42

(d) $(42)!$

(e) 1

15. Which one of the following statements is **False**?

(a) If f is differentiable then $\dfrac{d}{dx}\left(f(\sqrt{x})\right) = \dfrac{f'(x)}{2\sqrt{x}}$ for $x \neq 0$

(b) The derivative of a polynomial is a polynomial

(c) If $f(x) = (x^7 + x^2)^4$, then $f^{(29)}(x) = 0$

(d) $\dfrac{d}{dx}(\tan^2 x) = \dfrac{d}{dx}(\sec^2 x)$

(e) If $g(x) = x^6$, then $\lim\limits_{x\to 1} \dfrac{g(x) - g(1)}{x - 1} = 6$

16. If $x = \dfrac{\sin(y)}{\sin(1+y)}$ then $\dfrac{dy}{dx} =$

(a) $\dfrac{\sin^2(1+y)}{\sin(1)}$

(b) $\sin^2(1+y)$

(c) 1

(d) $\sin(1+y)$

(e) $\dfrac{\sin(y)}{\sin(1)}$

17. If $f(x) = (x-1)^2(x-2)^2(x-3)^2$, then $\dfrac{f'(0)}{f(0)} =$

(a) $\dfrac{-11}{3}$

(b) $\dfrac{-1}{3}$

(c) -3

(d) $\dfrac{-7}{3}$

(e) 0

18. Let $f(x) = 4x - \sin(2x)$ and $g(x) = f^{-1}(x)$, then $g'(0) =$

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{3}$

(d) 1

(e) 0

19. If $y = \cos^{-1}\left(\dfrac{1+2\cos\,x}{2+\cos\,x}\right)$ and $0 < x < \dfrac{\pi}{2}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{\sqrt{3}}{2+\cos x}$

(b) $\dfrac{\sqrt{\cos x}}{2+\sin x}$

(c) $\dfrac{\sqrt{3-\cos x}}{2}$

(d) $\dfrac{\sqrt{2+\cos x}}{2\sin x}$

(e) $\dfrac{\sqrt{\cos x}}{2-\sin x}$

20. If $y = mx + b$ is the equation of a line parallel to the line $y = (\ln\,2)x$ and tangent to the graph of $y = 2^{x+3}$, then $m + b =$

(a) $1 + 4\ln\,2$

(b) $1 - \ln\,2$

(c) $2 + 2\ln\,3$

(d) $\sqrt{2}$

(e) $3 + \sqrt{2}$