King Fahd University of Petroleum and Minerals Department of Mathematics and Statistics

> Math 101 Exam I Term 162 Sunday 12/03/2017 Net Time Allowed: 120 minutes

MASTER VERSION

$$1. \qquad \lim_{x \to \infty} \frac{\sqrt{9x^2 - 9}}{2x - 6} =$$

(a) $\frac{3}{2}$ (b) $\frac{2}{9}$ (c) -2 (d) 1 (e) $\frac{1}{3}$

2. The equations of the vertical asymptote(s) of $h(x) = \frac{x^2 - 1}{1 - x - 2x^2}$ is (are)

(a)
$$x = \frac{1}{2}$$

(b) $x = -1, x = \frac{1}{2}$
(c) $x = -\frac{1}{4}$
(d) $y = -\frac{1}{2}$
(e) $x = -\frac{1}{2}, x = \frac{1}{2}$

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- 3. The function $f(x) = \frac{\ln(2 + \cos e^x)}{x^2 4}$ is continuous for all x in the interval
 - (a) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
 - (b) $[0,\infty)$
 - (c) [-2,2]
 - (d) $(-\infty, 0) \cup (0, \infty)$

(e)
$$(-\infty, 0)$$

- 4. Let $f(x) = \frac{x^3 + 3x^2 9x 27}{x^3 9x}$. If *R* is the number of **re-movable** discontinuities of *f* and *I* is the number of **infinite** discontinuities of *f*, then
 - (a) R = 2 and I = 1
 - (b) R = 1 and I = 2
 - (c) R = 0 and I = 3
 - (d) R = 3 and I = 0
 - (e) R = 3 and I = 3

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5. If
$$\lim_{x \to 2} f(x) = 7$$
 and $\lim_{x \to 2} g(x) = 3$, then $\lim_{x \to 2} \frac{\sqrt{x + f(x)}}{|x - 2| - (g(x))^2} = 1$

(a)
$$-\frac{1}{3}$$

(b) $-\frac{2}{3}$
(c) -1
(d) 1

(e)
$$0$$

6.
$$\lim_{x \to \infty} \left[\tan^{-1} \left(\frac{1}{e^{-x} - 1} \right) \right] =$$

(a)
$$-\frac{\pi}{4}$$

(b) 0
(c) $\frac{\pi}{4}$
(d) $-\frac{\pi}{2}$
(e) $\frac{\pi}{2}$

7. If
$$f(x) = \begin{cases} ax + 2b & \text{for } x \leq 0\\ x^2 + 3a - b & \text{for } 0 < x \leq 2 \\ 3x - 5 & \text{for } x > 2 \end{cases}$$
 is a continuous function everywhere, then $f(1) =$

- (a) -2
- (b) -5
- (c) 3
- (d) 1
- (e) 0

8. The largest number $\delta > 0$, such that if $0 < |x - \mathbf{10}| < \delta$, then $|\sqrt{19 - x} - 3| < 1$, is

(You may use the graph of $y = \sqrt{19 - x}$)



(a) $\delta = 5$

- (b) $\delta = 7$
- (c) $\delta = 1$
- (d) $\delta = 3$
- (e) $\delta = 9$

9. If
$$\lim_{x \to 1} \frac{f(x) - 4}{x - 1} = 8$$
, then $\lim_{x \to 1} \frac{f(x)}{x + 1}$

- (a) equals 2
- (b) equals 6
- (c) equals 8
- (d) equals 4
- (e) Does not exist

10. If
$$f(x) = \begin{cases} x^3 \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$
, then

- (a) f'(0) = 0
- (b) f'(0) = 1
- (c) f'(0) = -1
- (d) $f'(0) = \frac{1}{2}$
- (e) f(x) is not differentiable at x = 0

11. Which one of the following statements is **TRUE**?

- (a) $e^x = 3 2x$ has one root in (0, 1)
- (b) If |f| is continuous at x = a, then f is continuous at x = a
- (c) If f is continuous at x = a, then f is differentiable at x = a
- (d) $x^2 = \cos x$ has no roots in $(-\pi, \pi)$
- (e) If f(x) = |x 6|, then f is not differentiable at x = 0

- 12. To prove that $\lim_{x\to 2} (2x-1) = 3$ by using the $\varepsilon \delta$ definition of the limit, we find that for given $\varepsilon = 0.002$, the largest possible value for δ that can be used is
 - (a) 0.001
 - (b) 0.05
 - (c) 0.002
 - (d) 0.003
 - (e) 0.02

13. The sum of all values of k, for which y = k is a horizontal asymptote to the graph of the function

$$f(x) = \begin{cases} \frac{2+\sqrt{x}}{2-\sqrt{x}} & \text{for} \quad x > 4\\ 1 & \text{for} \quad \sqrt[3]{\frac{3}{8}} \le x \le 4\\ \left(\frac{x^3+x-3}{8x^3-3}\right)^{1/3} & \text{for} \quad x < \sqrt[3]{\frac{3}{8}} \end{cases}$$
to

(a)
$$-\frac{1}{2}$$

(b) 1
(c) 0
(d) $\frac{3}{8}$
(e) $-\frac{1}{3}$

14.
$$\lim_{x \to 0^+} \sqrt{x} e^{\sin\left(\frac{\pi}{x}\right)} =$$

- (a) 0
- (b) 1
- (c) $\frac{1}{e}$
- (d) \sqrt{e}
- (e) -1

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- 15. Suppose f(x) is a differentiable function that satisfies the following f(x + y) = f(x) + f(y) + 2xy 1 for any real numbers x and y and $\lim_{x \to 0} \frac{f(x) 1}{x} = -2$. Then f'(x) =
 - (a) -2 + 2x
 - (b) -2
 - (c) -2 x
 - (d) 2x

(e)
$$-2x$$

16.
$$\lim_{t \to 4} \frac{\frac{1}{2} - \frac{1}{\sqrt{t}}}{t - 4} =$$

(a)
$$\frac{1}{16}$$

(b) $\frac{1}{4}$
(c) $\frac{1}{8}$
(d) $\frac{3}{14}$
(e) $\frac{1}{2}$

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17.
$$\lim_{x \to 1/3^+} \left[[3x] \right] =$$

(where [[x]] is the greatest integer less than or equal to x)

- (a) 1
- (b) 0
- (c) 2
- (d) -1
- (e) 3

18. Let a and b be real numbers. $\lim_{x \to -\infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx}) =$

(a)
$$\frac{1}{2}(b-a)$$

(b) $\frac{1}{\sqrt{2}}(a-b)$
(c) 0
(d) $\sqrt{a} - \sqrt{b}$

(e) $-\infty$

- 19. The equation of the tangent line to the curve $y = \frac{2}{1-3x}$ at the point with x-coordinate x = 0 is
 - (a) y = 6x + 2
 - (b) $y = \frac{1}{6}x + 2$
 - (c) y = 2x + 2
 - (d) $y = \frac{-1}{6}x + 2$

(e)
$$y = 3x - 3$$

20. If
$$f(x) = \begin{cases} \frac{1 - \cos x}{\sin x} & \text{for } x < 0\\ \frac{(1+x)^2 - 1}{x} & \text{for } x > 0 \end{cases}$$
, then

- (a) $\lim_{x \to 0} f(x)$ does not exist
- (b) $\lim_{x \to 0} f(x) = \infty$
- (c) $\lim_{x \to 0} f(x) = 1$
- (d) $\lim_{x \to 0} f(x) = 0$
- (e) $\lim_{x \to 0} f(x) = \frac{1}{2}$