King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Exam I
Term 162
Sunday 12/03/2017
Net Time Allowed: 120 minutes

MASTER VERSION

1. $\lim_{x\to\infty} \dfrac{\sqrt{9x^2-9}}{2x-6} =$

(a) $\dfrac{3}{2}$

(b) $\dfrac{2}{9}$

(c) -2

(d) 1

(e) $\dfrac{1}{3}$

2. The equations of the vertical asymptote(s) of $h(x) = \dfrac{x^2-1}{1-x-2x^2}$ is (are)

(a) $x = \dfrac{1}{2}$

(b) $x = -1\,,\ x = \dfrac{1}{2}$

(c) $x = -\dfrac{1}{4}$

(d) $y = -\dfrac{1}{2}$

(e) $x = -\dfrac{1}{2}\,,\ x = \dfrac{1}{2}$

3. The function $f(x) = \dfrac{\ln(2 + \cos e^x)}{x^2 - 4}$ is continuous for all x in the interval

(a) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(b) $[0, \infty)$

(c) $[-2, 2]$

(d) $(-\infty, 0) \cup (0, \infty)$

(e) $(-\infty, 0)$

4. Let $f(x) = \dfrac{x^3 + 3x^2 - 9x - 27}{x^3 - 9x}$. If R is the number of **removable** discontinuities of f and I is the number of **infinite** discontinuities of f, then

(a) $R = 2$ and $I = 1$

(b) $R = 1$ and $I = 2$

(c) $R = 0$ and $I = 3$

(d) $R = 3$ and $I = 0$

(e) $R = 3$ and $I = 3$

5. If $\lim_{x\to 2} f(x) = 7$ and $\lim_{x\to 2} g(x) = 3$, then $\lim_{x\to 2} \dfrac{\sqrt{x + f(x)}}{|x-2| - (g(x))^2} =$

(a) $-\dfrac{1}{3}$

(b) $-\dfrac{2}{3}$

(c) -1

(d) 1

(e) 0

6. $\lim_{x\to\infty} \left[\tan^{-1}\left(\dfrac{1}{e^{-x}-1}\right)\right] =$

(a) $-\dfrac{\pi}{4}$

(b) 0

(c) $\dfrac{\pi}{4}$

(d) $-\dfrac{\pi}{2}$

(e) $\dfrac{\pi}{2}$

7. If $f(x) = \begin{cases} ax + 2b & \text{for} \quad x \leq 0 \\ x^2 + 3a - b & \text{for} \quad 0 < x \leq 2 \\ 3x - 5 & \text{for} \quad x > 2 \end{cases}$ is a continuous function everywhere, then $f(1) =$

(a) -2

(b) -5

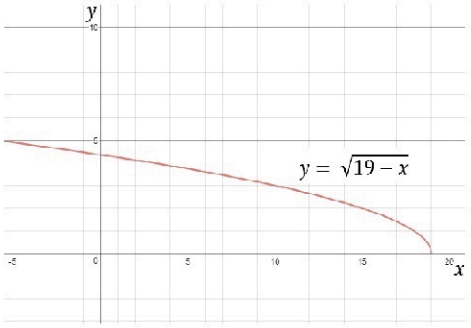
(c) 3

(d) 1

(e) 0

8. The largest number $\delta > 0$, such that if $0 < |x - 10| < \delta$, then $|\sqrt{19 - x} - 3| < 1$, is

(You may use the graph of $y = \sqrt{19 - x}$)



(a) $\delta = 5$

(b) $\delta = 7$

(c) $\delta = 1$

(d) $\delta = 3$

(e) $\delta = 9$

9. If $\lim_{x\to 1} \dfrac{f(x)-4}{x-1} = 8$, then $\lim_{x\to 1} \dfrac{f(x)}{x+1}$

(a) equals 2

(b) equals 6

(c) equals 8

(d) equals 4

(e) Does not exist

10. If $f(x) = \begin{cases} x^3 \sin \dfrac{1}{x} & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$, then

(a) $f'(0) = 0$

(b) $f'(0) = 1$

(c) $f'(0) = -1$

(d) $f'(0) = \dfrac{1}{2}$

(e) $f(x)$ is not differentiable at $x = 0$

11. Which one of the following statements is **TRUE**?

(a) $e^x = 3 - 2x$ has one root in $(0, 1)$

(b) If $|f|$ is continuous at $x = a$, then f is continuous at $x = a$

(c) If f is continuous at $x = a$, then f is differentiable at $x = a$

(d) $x^2 = \cos x$ has no roots in $(-\pi, \pi)$

(e) If $f(x) = |x - 6|$, then f is not differentiable at $x = 0$

12. To prove that $\lim_{x \to 2} (2x - 1) = 3$ by using the $\varepsilon - \delta$ definition of the limit, we find that for given $\varepsilon = 0.002$, the largest possible value for δ that can be used is

(a) 0.001

(b) 0.05

(c) 0.002

(d) 0.003

(e) 0.02

13. The sum of all values of k, for which $y = k$ is a horizontal asymptote to the graph of the function

$$f(x) = \begin{cases} \dfrac{2+\sqrt{x}}{2-\sqrt{x}} & \text{for} \quad x > 4 \\ 1 & \text{for} \quad \sqrt[3]{\dfrac{3}{8}} \le x \le 4 \\ \left(\dfrac{x^3+x-3}{8x^3-3}\right)^{1/3} & \text{for} \quad x < \sqrt[3]{\dfrac{3}{8}} \end{cases}$$

, equals to

(a) $-\dfrac{1}{2}$

(b) 1

(c) 0

(d) $\dfrac{3}{8}$

(e) $-\dfrac{1}{3}$

14. $\lim\limits_{x \to 0^+} \sqrt{x}\, e^{\sin\left(\frac{\pi}{x}\right)} =$

(a) 0

(b) 1

(c) $\dfrac{1}{e}$

(d) $\sqrt{e}$

(e) -1

15. Suppose $f(x)$ is a differentiable function that satisfies the following $f(x+y) = f(x) + f(y) + 2xy - 1$ for any real numbers x and y and $\lim_{x\to 0} \dfrac{f(x)-1}{x} = -2$. Then $f'(x) =$

(a) $-2 + 2x$

(b) -2

(c) $-2 - x$

(d) $2x$

(e) $-2x$

16. $\lim_{t\to 4} \dfrac{\frac{1}{2} - \frac{1}{\sqrt{t}}}{t-4} =$

(a) $\dfrac{1}{16}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{8}$

(d) $\dfrac{3}{14}$

(e) $\dfrac{1}{2}$

17. $\lim_{x \to 1/3^+} [[3x]] =$

(where $[[x]]$ is the greatest integer less than or equal to x)

(a) 1

(b) 0

(c) 2

(d) -1

(e) 3

18. Let a and b be real numbers. $\lim_{x \to -\infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx}) =$

(a) $\frac{1}{2}(b - a)$

(b) $\frac{1}{\sqrt{2}}(a - b)$

(c) 0

(d) $\sqrt{a} - \sqrt{b}$

(e) $-\infty$

19. The equation of the tangent line to the curve $y = \dfrac{2}{1-3x}$ at the point with x-coordinate $x = 0$ is

(a) $y = 6x + 2$

(b) $y = \dfrac{1}{6}x + 2$

(c) $y = 2x + 2$

(d) $y = \dfrac{-1}{6}x + 2$

(e) $y = 3x - 3$

20. If $f(x) = \begin{cases} \dfrac{1-\cos x}{\sin x} & \text{for} \quad x < 0 \\ \dfrac{(1+x)^2 - 1}{x} & \text{for} \quad x > 0 \end{cases}$, then

(a) $\lim\limits_{x\to 0} f(x)$ does not exist

(b) $\lim\limits_{x\to 0} f(x) = \infty$

(c) $\lim\limits_{x\to 0} f(x) = 1$

(d) $\lim\limits_{x\to 0} f(x) = 0$

(e) $\lim\limits_{x\to 0} f(x) = \dfrac{1}{2}$