

# King Fahd University of Petroleum & Minerals

# First Major Examination

Faculty: Science	Department: Mathematics
Semester: 171	Course Name: Actuarial Risk & Credibility Theory
Instructor: Abedalhay Elmughrabi	Course No: AS 483
Exam Date: March 12 <sup>th</sup> , 2017	Exam Time: 02:10 PM – 03:40 PM (90 Minuets)

Student Name:	ID No.:
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Question No.	Question Full Marks	Question Obtained Marks
1	10 points	
2	10 points	
3	10 points	
4	10 points	
5	10 points	
6	10 points	
7	10 points	
8	10 points	
9	10 points	
10	10 points	
Total	100	Obtained Total:



## Exam Instructions

- 1. Fill in all information required.
- 2. The exam is composed of **10** questions.
- 3. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
- 4. Calculators cannot be exchanged during the examination.
- 5. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
- 6. Cell phones must be turned off and placed under your bench facedown.
- 7. No questions are allowed during the exam.
- 8. All material related to the course should be put away
- 9. Final correct answers have significant weights
- 10. Answers without calculations/steps will receive zero marks.
- 11. Be clean, neat and tidy, else your work may not be marked
- 12. Students must not communicate with one another in any manner whatsoever during the examination.

# GOOD LUCK



#### **Questions 1 (10 Points):**

The random variables X and Y have joint density function

 $f(x,y) = e^{-2x-\frac{y}{2}}, \quad 0 < x < \infty, \quad 0 < y < \infty$ Determine the mean excess loss function for the marginal distribution of X evaluated at X=4.



#### **Questions 2 (10 Points):**

The loss severity random variable X follows the exponential distribution with mean 10,000. Determine the coefficient of variation of the excess loss variable  $Y = (X - 30,000)_+$ 



## Questions 3 (10 Points):

X is a random variable for losses. X follows a beta distribution with  $\theta$ =1000, a=2, b=1. Calculate TVaR<sub>0.90</sub>(X)



### **Questions 4 (10 Points):**

Y is a mixture of two exponential distributions  $f_Y(y) = \frac{1}{2}e^{-y} + \frac{1}{6}e^{-y/3}$ . The random variable Z=2Y is a mixture of two exponentials. What are the means of those two exponential distributions?



## **Questions 5 (10 Points):**

X has an exponential distribution with mean  $\theta$  and  $\theta$  has a uniform (0,100). Y has a Uniform distribution (0, $\alpha$ ) and  $\alpha$  has an exponential distribution with mean 100. Find  $\frac{Var(X)}{Var(Y)}$ 



#### **Questions 6 (10 Points):**

Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter  $\lambda$ , where  $\lambda$  follows the gamma distribution with mean 3 and variance 3. Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.



# Questions 7 (10 Points):

For a discrete distribution, you are given

• 
$$p_0 = 0.8$$

• 
$$p_k = \frac{p_{k-1}}{4k}$$
 for k > 1

Calculate the mean and variance of the distribution?



#### **Questions 8 (10 Points):**

- In 1998, claim sizes follow a Pareto distribution with parameters  $\theta$  (unknown) and  $\alpha=2$ .
- Inflation of 6% affects all claims uniformly from 1998 to 1999.
- r is the ratio of the proportion of claims that exceed d in 1999 to the proportion of claims that exceed d in 1998.

Determine the limit of r as d goes to infinity.



## **Questions 9 (10 Points):**

The random variable N follows a zero modified Poisson distribution. You are given:

P(N=1)=0.25 P(N=2)=0.1

Calculate the probability of 0?



## Questions 10 (10 Points):

Let X have a Pareto distribution with parameters  $\alpha$  and  $\theta$ . Let  $Y = \ln(1 + \frac{x}{\theta})$ Determine the name of the distribution of Y and its parameter(s).