KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
STAT416 : Stochastic Processes for Actuaries (161)
Third Exam **Thursday December 29, 2016**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	14	
Two	12	
Three	12	
Four	12	
Five	15	
Six	20	
Total	85	

Question.1 (**4+4+3+3=14-Points**)
Define the following:

(a) Brownian motion.

(b) Geometric Brownian motion.

(c) Gaussian Process $\{X(t); t \geq 0\}$.

(d) Integrated Brownian motion.

Question.2 **(6+6=12-Points)**

Let $\{X(t); t \geq 0\}$ be a standard Brownian motion.

(a) Find $P\{X(1) + X(2) > 2\}$.

(b) If $X(3) = 5$, find the probability that the process is at least 6 at $t = 7$.

Question.3 **(12-Points)**

Let $\{B(t); t \geq 0\}$ be a standard Brownian motion, and $\{X(t)\}$ be a geometric Brownian motion. Then, for $0 \leq s \leq t$, show that: $cov\left(X(s), X(t)\right) = e^{\left(\frac{3s+t}{2}\right)} - e^{\left(\frac{s+t}{2}\right)}$

Question.4 **(3+3+2+4=12-Points)**

The price of a commodity moves according to a Brownian motion $X(t) = \sigma B(t) + \mu t$, with $\sigma^2 = 4$ and a drift $\mu = -5$, where $B(t)$ is a standard Brownian motion.

(a) Find $E\left(X(9) - X(8)\right)$.

(b) Find $Var\left(X(9) - X(8)\right)$.

(c) What is the distribution of $X(9) - X(8)$?

(d) Given that the price is 4 at $t = 8$, what is the probability that the price is below 1 at $t = 9$?

Question 5. **(8+7=15-Points)**

Let X be a random variable having the following pdf:

$$f(x) = \begin{cases} 2-2x & ,0 < x < 1 \\ 0 & ,Otherwise \end{cases}$$

(a) Explain a procedure that applies the inverse transformation method to generate an observation from this distribution using a uniform random number, say U, where $U \sim U(0,1)$. Clearly, specify the generated X in terms of U.

(b) Use the following uniform random numbers to estimate the **mean** of this distribution. $u_1 = 0.82$, $u_2 = 0.06$, $u_3 = 0.68$, $u_4 = 0.44$, and $u_5 = 0.31$. (round your answer to four decimal places)

Question 6. (**6+14=20-Points**)

Let X be a binomial random variable with $n = 3$ and $p = 0.90$.

(a) Use the following uniform random numbers to generate one observation from the distribution of X. $u_1 = 0.99$, $u_2 = 0.32$, and $u_3 = 0.76$. (Clearly, write all your steps).

(b) Use only $u_1 = 0.99$ to generate one observation from the distribution of X. (Clearly, write all your steps).