KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
STAT416 : Stochastic Processes for Actuaries (161)
Second Exam **Wednesday December 4, 2016**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	10	
Two	16	
Three	20	
Four	14	
Five	10	
Six	15	
Seven	15	
Total	100	

Question.1 (**2+2+2+2+2=10-Points**)

Answer the following:

(a) Define a birth and death process.

(b) When we say that a continuous time Markov chain has independent increments?

(c) For an $M/M/1$ queueing system, what is the meaning of $\pi_0 > 0.90$?

(d) Explain the meaning of all numbers in the queueing system $M/M/3/k/20$.

(e) Define a pure birth process.

Question.2 **(6+4+2+4=16-Points)**

A statistic instructor can not receive more than two students at the same time in his office. Usually, one day before the exam, students arrive according to a Poisson process with rate $\lambda = 3$ per hour to ask questions. The instructor helps the students one at a time. There is one chair in his office where the student can wait for his turn. If a student arrives when two other students are already in the instructor's office, then he must come back later. Assume the that the instructor takes to answer any student is, independently from any other one, distributed exponentially at rate 15 minutes. If we consider only the days preceding an exam, then

(a) Calculate the proportion of time, on the long run, where the instructor is not busy answering questions.

(b) Find the average number of students in the instructor's office ?

(c) Assume there is a student asking the instructor questions, what is the average waiting time for the next student to get his turn?

(d) If the instructor spends twice more time, on average, with each student, what would be the answer to part (a)?

Question.3 **(17+3=20-Points)**

A job shop consists of three machines and two repairmen. The amount of time a machine works before breaking down is exponentially distributed with mean 10. If the amount of time it tales a single repairman to fix a machine is exponentially distributed with mean 4, then

(a) What is the average number of machines not in use?

(b) What proportion of time are both repairmen busy?

Question.4 **(3+3+5+3=14-Points)**

For a queueing system with states $\{0, 1, 2\}$, and rate matrix $\boldsymbol{Q} = \begin{pmatrix} -2 & 2 & 0 \\ 1 & -2 & 1 \\ 0 & 2 & -2 \end{pmatrix}$

(a) Write down the forward differential equation for $P'_{22}(t)$. (Do not solve).

(b) Write down the backward differential equation for $P'_{11}(t)$. (Do not solve).

(c) Find the limiting probability vector for this system.

(d) Find the expected number of arrivals in this system.

Question 5. **(10-Points)**

For an $M/M/1$ queueing system, it can be shown that $\pi_n = \rho^n(1-\rho)$, $n = 0, 1, 2, \ldots$, where $\rho = \frac{\lambda}{\mu}$. Assume that $\rho < 1$, show that the expected length of the queue is given by $\frac{\lambda^2}{\mu(\mu-\lambda)}$.

Question 6. (**3+3+3+3+3=15-Points**)

Students arrive at the campus post office according to a Poisson process with an average rate of one student every 4 minutes. The time required to serve each student is exponentially distributed with a mean of 3 minutes. There is only one postal worker at the counter, and any arriving student that finds the worker busy joins a queue.

(a) Specify the type of queueing system and find the corresponding parameters

(b) What is the probability that an arriving student has to wait?

(c) What is the probability that an arriving student waits less that 2 minutes?

(d) What is the average waiting time of an arbitrary student?

(d) What is the mean number of students who are waiting at the postal office?

Question 7. (**8+2+3+2=15-Points**)

Customers arrive at a two-server station in accordance with rate of two per hour. Arrivals finding server 1 free begin service with that server. Arrivals finding server 1 busy and server 2 free begin service with server 2. Arrivals finding both servers busy are lost. When a customer is served by server 1, he then either enter the service with server 2 if 2 is free or departs the system if 2 is busy. A customer completing service at server 2 departs the system. The service times at server 1 and server 2 are exponential random variables with respective rates four and six per hour.

(a) Write down the four balance equations. (Do not solve)

If the limiting probability vector of being in the states $(0,0),(0,1),(1,0),(1,1)$ is $\boldsymbol{\pi} = \left(\frac{1}{2}, \frac{1}{6}, \frac{1}{4}, \frac{1}{12}\right)$, answer the following:

(b) find the fraction of customers do not enter the system.

(c) What is the average number of having exactly one customer in the system?

(d) What is the provability that the system is busy?