King Fahd University Of Petroleum & Minerals
Department Of Mathematics And Statistics
STAT416 : Stochastic Processes for Actuaries (161)
First Exam **Thursday November 3, 2016**
Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	12	
Two	18	
Three	12	
Four	11	
Five	13	
Six	14	
Total	80	

Question.1 (**2+2+2+4+2=12-Points**) Define the following:

(a) Markov Chain:

(b) Recurrent state:

(c) Counting Process $\{N(t), t \geq 0\}$

(d) Period of a state:

(e) Poisson Process:

Question .2 **(2+8+8=18-Points)**

Suppose that a Markov chain has two states {1,2}. and $\boldsymbol{P}^n = \frac{1}{6}\begin{pmatrix} 5+(0.4)^n & 1-(0.4)^n \\ 5-5(0.4)^n & 1+5(0.4)^n \end{pmatrix}$

(a) Write the transition probability matrix $\boldsymbol{P}$

(b) If $P(X_0 = 1) = P(X_0 = 2)$, calculate $E(X_4)$

(c) Find the probability that the process is in state 2 on step 5 and on step 6, given that it was in state 1 on step 1.

Question.3 **(2+2+3+5=12-Points)**

For a branching process with offsprings distribution given by: $b_0 = \frac{1}{6}, b_1 = \frac{1}{2}, b_3 = \frac{1}{3},$

(a.) Write down the probability generating function.

(b.) Find $P\left(X_1 = 1 | X_0 = 1\right)$.

(c.) Find the probability of extinction (π_0) given that the process starts with one individual only.

Question.4 **(3+3+5=11-Points)**

A Markov chain with four states $\{0, 1, 2, 3\}$ and a transition probability matrix given by:

$$\boldsymbol{P} = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.5 & 0.25 & 0 \\ 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix}$$

(a) Determine the classes of this Markov Chain. Is it irreducible? Why?

(b) Find the period of each state. Is it a periodic or aperiodic?

(b) Find the the stationary vector.

Question 5. **(5+6+2=13-Points)**

Assume the intensity functions of two independent nonhomogeneous Poisson process are given by:

$$\lambda(t) = \begin{cases} 2 & ,t \in (1,2],(3,4] \\ 3 & ,t \in (0,1],(2,3] \end{cases}$$

$$\mu(t) = \begin{cases} 5 & ,t \in (1,2],(3,4] \\ 1 & ,t \in (0,1],(2,3] \end{cases}$$

(a) Find the probability that the number of events in the first process in $(1.5, 3]$ is less than 2.

(b) Find the probability that the total number of events in $[0, 2]$ is exactly 3.

(b) Given that the total number of events in $(0, 2]$ is 3, what is the probability that all events occurred from the second process?

Question 6. (**14-Points**)

Suppose that the number of calls per hour to an answering service follows a Poisson process with rate λ. Find the probability that there are two calls in the interval $(0, 2]$ and three calls in the interval $(1, 4]$ received by the answering service.