KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
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STAT 310: Linear Regression
Semester 161
Quiz 2 (Mathematical)
Tuesday December 20, 2016
3:00 pm

Name: ID #:

Q.No.1:- (2+2+3 = 7 points) Suppose that we want to fit the model $y = \beta x + \epsilon$ using weighted least squares. Assume that the observations are uncorrelated but have unequal variance.

(a) Find a general formula for the weighted least-square estimator of β. Simplify as much as possible.

(b) What is the variance of the weighted least-square estimator derived in part (a)?

(c) Suppose that $Var(y_i) = cx_i$ i.e. the variance is proportional to the corresponding x_i. Using the result of parts (a) and (b), find the weighted least-square estimator of β and the variance of this estimator. (Hint: The weights (w_i) should be reciprocal of x_i)

Q.No.2:- (4 points) Suppose that the studentized PRESSresidual $t_i = \frac{e_{(i)}}{\sqrt{Var(e_{(i)})}}$ where $e_{(i)} = \frac{e_i}{1-h_{ii}}$.

Mathematically show that $t_i = \frac{e_i}{\sqrt{\sigma^2(1-h_{ii})}}$.

Q.No.3:- (4 points) Consider the multiple linear regression model $y = X\beta + \varepsilon$. Show that the least-square estimator of β (i.e. $\hat{\beta} = (X'X)^{-1}X'y$) can be written as $\hat{\beta} = \beta + X^{-1}H\varepsilon$ where $H = X(X'X)^{-1}X'$.