Department of Mathematics and Statistics
Semester 161

STAT302 Final Exam Saturday January 14, 2017

Name: _____ ID #: _____

Instructions:

- Justify your work
- State theorems and results you are using
- Show all details
- In hypothesis testing problems, write down the hypotheses, the rejection region, the decision and the conclusion.

Question	Marks	Marks Obtained
1	6	
2	8	
3	11	
4	5	
5	5	
6	8	
7	7	
Total	50	

1) An auditor selected a random sample of 9 accounts from all accounts receivable of a given firm. The amounts in millions of riyals are assumed to follow a normal distribution with standard deviation of 4 million.
   a) Find the probability that the sample mean will be within 2 million riyals of the mean of all accounts. *(3 pts.)*

   b) If the auditor would like the sample mean to be within 1 million riyals, with probability 0.90. How many accounts must he sample in order to ensure this degree of accuracy? *(3 rpts.)*

2) If Y has a probability density function

$$f_Y(y) = \begin{cases} \frac{2(\theta - y)}{\theta^2}, & 0 < y < \theta \\ 0 & otherwise \end{cases}$$

a) Show that Y/θ is a pivotal quantity. *(3 pts.)*

b) Use this pivotal quantity to find a 90% upper confidence limit for θ. *(5 pts.)*

3) Let $Y_1, \cdots, Y_n$ be a random sample from a uniform distribution on the interval $[\theta, \theta + 1]$, and consider the two estimators $\hat{\theta}_1 = \bar{Y} - \frac{1}{2}$ and $\hat{\theta}_2 = \frac{Y_1 + Y_2}{2} - \frac{1}{2}$.

a) Is $\hat{\theta}_1$ unbiased? Is it consistent? *(4 pts.)*

b) Is $\hat{\theta}_2$ unbiased? Is it consistnet? *(3 pts.)*

c) What is the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$? *(2 pts.)*

d) Based on the relative efficiency computed above, which estimator is better? *(2 pts.)*

4) Let $Y_1, \cdots, Y_n$ be a random sample from the exponential distribution

$$f_Y(y) = \begin{cases} \theta e^{-\theta y}, & y > 0 \\ 0 & otherwise \end{cases}$$

a) Find the maximum likelihood estimator of θ. *(3 pts.)*

b) Find the maximum likelihood estimator of $1/\theta$. *(2 pts.)*

5) Suppose Y is a single observation from a population with probability density

$$f_Y(y) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Find the most powerful test with significance level 0.05 for testing

$H_0: \theta = 3 \quad vs \quad H_a: \theta = 2$ *(5 pts.)*

6) In an $r \times c$ contingency table, we can have two conceptually different problems that are analyzed the same way, namely using a chi-square test. Using θ_{ij} to denote the probability of an outcome falling in the i^{th} row and j^{th} column. Explain

a) the two problems *(3 pts.)*

b) The hypotheses in each of them *(2 pts.)*

c) The test statistic *(1 pt)*

d) The details of how to obtain the number of degrees of freedom. *(2 pts.)*

7) Each day a baker bakes three cakes, and those not sold on the same day are given away to the poor. Use the data shown in the following table to test at the 0.05 significance level whether they may be looked upon as values of a binomial random variable?

Number of Cakes Sold	0	1	2	3
Number of Days	1	16	55	228

(7 pts.)