KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS

STAT 302 Exam #2

Name:_____ ID#: _____

1) Assume *Y* has a binomial distribution with parameters *n* and *p*, and consider the estimator $\hat{p} = \frac{Y+1}{n+2}$.

   i) Is $\hat{p}$ unbiased? If not what is its bias?

   ii) Derive *MSE*($\hat{p}$).

2) If $Y_1, \cdots, Y_n$ are independent and identically distributed variables from a Pareto distribution with parameters α and β, and with a density

$$f_Y(y) = \begin{cases} \alpha\beta^{\alpha} y^{-(\alpha+1)}, & y \geq \beta \\ 0 & otherwise \end{cases}.$$

If β is known, find a sufficient statistic for α.

3) If Y has a normal distribution with mean 0 and variance σ^2, find a $100(1-\alpha)\%$ confidence interval for σ^2 using the pivotal quantity method.

4) If $Y_1, \cdots, Y_n$ is random sample from a distribution with probability density

$$f_Y(y) = \begin{cases} \theta(\theta+1)y^{\theta}, & 0 < y < 1, \quad \theta > -1 \\ 0 & otherwise \end{cases}$$

i) Find an estimator of θ by the method of moments.

ii) Is this estimator consistent? Justify your answer.

iii) Is it MVUE? Justify your answer.

iv) Find the MLE for θ.

5) If $Y_1, \cdots, Y_n$ is random sample from a normal population with mean μ and variance σ^2 . Assuming $n = 2k$ for some integer k, and consider the estimator

$$\hat{\sigma}^2 = \frac{1}{2k}\sum_{i=1}^{k}(Y_{2i} - Y_{2i-1})^2.$$

i) Show that $\hat{\sigma}^2$ is an unbiased estimator for σ^2.

ii) Show that $\hat{\sigma}^2$ is a consistent estimator for σ^2.