KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 161

STAT 213 STATISTICS METHODS FOR ACTUARIES

Wednesday Nov. 9, 2016

Name: ____ ID #:___________________________

Important Note:

- Show all your work including formulas, intermediate steps and final answer

Question No	Full Marks	Marks Obtained
1	9	
2	9	
3	10	
4	12	
5	15	
6	5	
Total	60	

Q1: An insurance company insures a large number of homes. The insured value, X of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = 3x^{-4} \quad x > 1$$

1. Find the probability that a randomly selected home is insured for at least 1.5. (*2 pts.*)

2. Given that a randomly selected home insured for at least 1.5, what is the probability that it is insured for less than 2? (*5 pts.*)

3. Find the expected value for the random variable X. (*2 pts.*)

Q2: A probability distribution of the claim size for an auto insurance policy is given in the table below:

claim size	20	30	40	50	60	70	80
probability	0.15	0.1	0.05	0.2	0.1	0.1	0.3

1. Find the mean claim size. (*3 pts.*)

2. Find the standard deviation of the claim size. (*3 pts.*)

3. What is the percentage of the claims are within one standard deviation of the mean claim size. (*3 pts.*)

Q3: Suppose that only 60% of all drivers in Al-Khobar wear seat belt. Assume that X is a random variable denote the number of drivers that wear seat belt

1. If a random sample of size n drivers is selected, state the exact distribution of X, state any assumptions if needed. (*3 pts.*)

2. A random sample of size 10 drivers is selected, find the probability that less than two drivers wear seat belt. (*2 pts.*)

3. A random sample of size 500 drivers is selected, approximate the probability that X is between 270 and 320 inclusive. (*5 pts.*)

Q4: At a computer manufacturing company, the actual size of computer ships is normally distributed with mean of 1 centimeter and standard deviation of 0.1 centimeter.

1. A computer chip is randomly selected, find the probability that the actual size exceeds 0.9 centimeter. (*2 pts.*)

2. What is the 90^{th} percentile size? (*2 pts.*)

3. A random sample of size 12 computer chips is taken.

   a. What is the probability that the sample mean will be between 0.99 and 1.01 centimeters? (*5 pts.*)

   b. Above what value do 2.5% of the sample means fall? (*3 pts.*)

Q5: Customers arrive in a certain shop at mean rate 5 per hour.

1. Find the probability that exactly two customers will arrive in a given hour. (*2 pts.*)

2. Find the probability that at least one customer will arrive within a 10 – minute interval. (*2 pts.*)

3. What is the expected waiting time (in minutes) before the first arrival? (*2 pts.*)

4. Find the probability that the shopkeeper will wait more than 5 minutes for the arrival of the first customer. (*2 pts.*)

5. What is the median time (in minutes) until the first arrival? (*2 pts.*)

6. Find the probability that the average time between every two successive customers in a group of 36 will not exceed 8 minutes. (*5 pts.*)

Q6: (*5 pts.*) Suppose that he random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

Suppose that a random sample of size 25 observations is selected from this distribution. What is the approximate distribution of $\bar{X} + \frac{1}{2}$? Find the mean and the standard deviation of this quantity.

Formula Sheet

Descriptive Statistics

- $P(X = x) = C_x^n \pi^x (1-\pi)^{n-x}, \quad x = 0, 1, 2, \dots, n; \quad \mu = n\pi \;\&\; \sigma = \sqrt{n\pi(1-\pi)}$
- $P(X = x) = \frac{C_x^K C_{n-x}^{N-K}}{C_x^N}, \;\; x = max\{0, n+k-N\} \; to \; min\{k, n\};$

$$\mu = n\frac{k}{N} \;\&\; \sigma = \sqrt{n\frac{k}{N}\left(1-\frac{k}{N}\right)}\sqrt{\frac{N-n}{N-1}}$$

- $P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, \;\; x = 0, 1, 2, \dots; \quad \mu = \lambda t \quad \& \quad \sigma = \sqrt{\lambda t}$
- $E(X) = \sum x\, P(X = x) \;\; or \;\; E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$
- $E(X^2) = \sum x^2\, P(X = x) \;\; or \;\; E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$
- $\sigma^2 = E(X^2) - \big(E(X)\big)^2$
- $f(x) = \frac{1}{x_n - x_1}, \quad x_1 \le x \le x_n; \quad \mu = \frac{x_n + x_1}{2} \quad \& \quad \sigma = \sqrt{\frac{(x_n - x_1)^2}{12}}$
- $f(x) = \lambda e^{-\lambda x}, \quad x > 0; \quad \mu = \frac{1}{\lambda} \quad \& \quad \sigma = \frac{1}{\lambda}$
- If $X \sim normal \; with \; \mu \;\&\; \sigma$, then

$$Z = \frac{X - \mu}{\sigma} \sim normal \; with \; 0 \;\&\; 1$$

and

$$Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim normal \; with \; 0 \;\&\; 1$$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \;\; P(B) > 0$