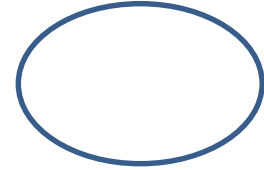


KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 161

STAT 213 STATISTICS METHODS FOR ACTUARIES

Wednesday Nov. 9, 2016



Name: _____ ID #: _____

Important Note:

- Show all your work including formulas, intermediate steps and final answer

Question No	Full Marks	Marks Obtained
1	9	
2	9	
3	10	
4	12	
5	15	
6	5	
Total	60	

Q1: An insurance company insures a large number of homes. The insured value, X of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = 3x^{-4} \quad x > 1$$

1. Find the probability that a randomly selected home is insured for at least 1.5. (2 pts.)

2. Given that a randomly selected home insured for at least 1.5, what is the probability that it is insured for less than 2? (5 pts.)

3. Find the expected value for the random variable X . (2 pts.)

Q2: A probability distribution of the claim size for an auto insurance policy is given in the table below:

claim size	20	30	40	50	60	70	80
probability	0.15	0.1	0.05	0.2	0.1	0.1	0.3

1. Find the mean claim size. (3 pts.)

2. Find the standard deviation of the claim size. (3 pts.)

3. What is the percentage of the claims are within one standard deviation of the mean claim size. (3 pts.)

Q3: Suppose that only 60% of all drivers in Al-Khobar wear seat belt. Assume that X is a random variable denote the number of drivers that wear seat belt

1. If a random sample of size n drivers is selected, state the exact distribution of X , state any assumptions if needed. (3 pts.)
2. A random sample of size 10 drivers is selected, find the probability that less than two drivers wear seat belt. (2 pts.)
3. A random sample of size 500 drivers is selected, approximate the probability that X is between 270 and 320 inclusive. (5 pts.)

Q4: At a computer manufacturing company, the actual size of computer chips is normally distributed with mean of 1 centimeter and standard deviation of 0.1 centimeter.

1. A computer chip is randomly selected, find the probability that the actual size exceeds 0.9 centimeter. (2 pts.)

2. What is the 90th percentile size? (2 pts.)

3. A random sample of size 12 computer chips is taken.

- a. What is the probability that the sample mean will be between 0.99 and 1.01 centimeters? (5 pts.)

- b. Above what value do 2.5% of the sample means fall? (3 pts.)

Q5: Customers arrive in a certain shop at mean rate 5 per hour.

1. Find the probability that exactly two customers will arrive in a given hour. (2 pts.)

2. Find the probability that at least one customer will arrive within a 10 – minute interval. (2 pts.)

3. What is the expected waiting time (in minutes) before the first arrival? (2 pts.)

4. Find the probability that the shopkeeper will wait more than 5 minutes for the arrival of the first customer. (2 pts.)

5. What is the median time (in minutes) until the first arrival? (2 pts.)

6. Find the probability that the average time between every two successive customers in a group of 36 will not exceed 8 minutes. (5 pts.)

Q6: (5 pts.) Suppose that the random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that a random sample of size 25 observations is selected from this distribution. What is the approximate distribution of $\bar{X} + \frac{1}{2}$? Find the mean and the standard deviation of this quantity.

Formula Sheet

Descriptive Statistics

- $P(X = x) = C_x^n \pi^x (1 - \pi)^{n-x}$, $x = 0, 1, 2, \dots, n$; $\mu = n\pi$ & $\sigma = \sqrt{n\pi(1 - \pi)}$
- $P(X = x) = \frac{C_x^k C_{n-x}^{N-k}}{C_x^N}$, $x = \max\{0, n + k - N\}$ to $\min\{k, n\}$;
 $\mu = n \frac{k}{N}$ & $\sigma = \sqrt{n \frac{k}{N} \left(1 - \frac{k}{N}\right) \frac{N-n}{N-1}}$
- $P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$, $x = 0, 1, 2, \dots$; $\mu = \lambda t$ & $\sigma = \sqrt{\lambda t}$
- $E(X) = \sum x P(X = x)$ or $E(X) = \int_{-\infty}^{\infty} x f(x) dx$
- $E(X^2) = \sum x^2 P(X = x)$ or $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$
- $\sigma^2 = E(X^2) - (E(X))^2$
- $f(x) = \frac{1}{x_n - x_1}$, $x_1 \leq x \leq x_n$; $\mu = \frac{x_n + x_1}{2}$ & $\sigma = \sqrt{\frac{(x_n - x_1)^2}{12}}$
- $f(x) = \lambda e^{-\lambda x}$, $x > 0$; $\mu = \frac{1}{\lambda}$ & $\sigma = \frac{1}{\lambda}$
- If $X \sim \text{normal}$ with μ & σ , then

$$Z = \frac{X - \mu}{\sigma} \sim \text{normal with } 0 \text{ \& } 1$$

and

$$Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \text{normal with } 0 \text{ \& } 1$$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A | B) = \frac{P(A \cap B)}{P(B)}$, $P(B) > 0$