King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 605: Asymptotic Expansions and Perturbation Methods
Final Exam-Part II: Take home
Instructor: Prof. B. Chanane

Name:_____ ID:_____

Due Monday 16 January 2017 at 11AM

Problem 1 *The purpose is to find a uniformly valid two-term approximation to the solution of the problem*

$$\begin{cases} \frac{d^2y}{dt^2} + \frac{dy}{dt} + \epsilon y^2 = 0 \\ y(0) = 1,\ y'(0) = 0 \end{cases}$$

for $0 < \epsilon << 1$.
1) Show that a regular asymptotic expansion as solution gives rise to secular terms. Explain.
2) Use the two-scale method with

$$\begin{aligned} t_1 &= \epsilon t \\ t_2 &= t\left(1 + p_1\epsilon^2 + p_2\epsilon^3 + \cdots\right) \end{aligned}$$

to find a uniformly valid two-term asymptotic expansion to the solution of the problem. Here, p_1, p_2,$\cdots$ *are additional parameters to be determined in the process of finding the approximation.*
3) Draw on the same scale the numerical solution as well as the approximate solution for $\epsilon = 0.1$, *and* $\epsilon = 0.01$ *and*
4) Print the listing of the program developed for this purpose.

Problem 2 *Use the method of matched asymptotic expansions to find a uniformly valid approximation to the solution of the problem*

$$\begin{cases} \epsilon\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0 \\ y(0) = 1,\ y(1) = 0 \end{cases}$$

(provide all details)
3) draw on the same scale the numerical solution as well as the inner, the outer solutions and the approximate solution for $\epsilon = 0.2$, $\epsilon = 0.1$ *and* $\epsilon = 0.01$ *and*
4) Print the listing of the program developed for this purpose.